The Montessori Math Curriculum
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Fractions

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Requirements for Mathematics Album

Introduction: - A brief history (cultural perspective) of mathematics

* Maria Montessori’s concept of the Mathematical Mind
* Montessori’s points of Mathematical Theory:

∀ order

∀ manipulation

∀ concrete to abstract

∀ isolation

∀ repetition

∀ exactness

Ways in which other Montessori curriculum areas prepare the child for math. You may also wish to relate your own experience in learning math.

You are also responsible to tab and organize your lessons and illustrations within the album in a clear and sequential manner as outlined on the sections cover sheets. The curriculum is divided into the following five main areas:

* Numeration (Numbers 1 – 10)
* Linear Counting
* Decimal System
* Memorization
* Passage to Abstraction

Also to include:

* Fractions
* Geometry
* Reflective Journal entries (compiled during internship phase)\*
* Other additional information and resources

\*These will be reflective writings based on the presentation of various materials and your observations and ‘reflections’ of the presentation and child’s experience with the material.

Your album must contain appropriate presentation illustrations. It is not a requirement to include illustrations of variations and extensions, or support material. You may choose to include photographs to further illustrate material presentations or to personalize your album.

Name of Activity: Area: General:

Specific:

Materials:

Aims: Direct: Indirect:

Preparation: Age:

Presentation of the Lesson: Work of the Teacher:

Points of Emphasis: Language:

Work of the Child:

Points of Interest Points of Consciousness: Control of Error: Variations:

Extensions: Source:

### Math Curriculum Course Overview

**Numeration**

This initial stage of the math curriculum is designed to first help the child to better understand the experience of quantity. This will lead to an understanding of the concept of quantity as universal. Secondly, these numeration materials will assist the child in recognizing the numerical symbols. And, finally, the child will experience the association of quantities with the appropriate symbolic numeral.

These skills and understandings will take place through the ‘isolation’ of difficulties - the child is first presented with fixed quantities, then with unsequenced numerals, then with an association of unsequenced numerals to fixed quantities, then with the association of sequenced numerals with lose quantities and finally, the child is prepared to create his own sets of quantities and sequenced numerals.

The child eventually can identify a quantity (set) by counting its members and place the appropriate symbolic numeral with it.

**Linear Counting**

In this area of the math curriculum, the child is given many materials with which to count in a linear fashion. The child counts beads as she re-experiences the concept of ten; the child counts teens and tens and even places numerals in sequence on a board that goes up to 100.

The child also has initial introduction to the concepts of squaring and cubing with chains that can be stretched into long lines. In accomplishing this type of exercise, the child is also learning how to ‘skip’ count.

### The Decimal System

The child will work in our base ten system of math, and this area of the curriculum allows the child to be introduced to how this system operates mathematically. The child learns the base ten concepts and operations through the work with the golden bead materials. The child is exposed to the quantities and symbols from 1 - 9,999 and has an initial, experiential process with units, tens, hundreds and thousands.

The child has further work in associating the quantities with the symbols, both in working with the golden bead layout, and with the exercises for the composition of numbers.

The child is also introduced to equivalencies, i.e., that 10 tens is the same as 100, In addition, the child is exposed to the concept of exchange: that 10 tens can be “exchanged” (traded) for one hundred.

These exercises are laying the groundwork for the four mathematical operations of addition, multiplication, subtraction and division.

### Decimal System: Operations

The child has now had the necessary groundwork for entering into the operational work with the golden bead materials. The child will now experience the process of “putting things (numbers) together” (addition); of a special kind of addition of “putting the same number of things together” or by “taking the same number more than once” (multiplication); of “taking away” (subtraction), and finally, of taking a quantity and “separating it into an equal number of parts” (division).

The child will accomplish these operations first by using the golden bead materials with which she has already become familiar with through the base ten system exercises.

The child will first experience “static” operations: math problems that do not require "carrying” or “exchanging” (also referred to as “regrouping”) from one place value to another. The child will then proceed to “dynamic” operations which will challenge the child to add, multiply, subtract and divide with quantities that must be “exchanged” in order to solve the problem. At this point, the child will still be working on a very concrete level with the use of the golden bead “bank materials” which allow the child to feel and visualize the quantities.

### Memorization- Leading to Abstraction

Now for the first time, the quantities will no longer have a sensorial and qualitative direct representation to the numerical symbol. The child will have to be able to mentally represent the association between the quantity and the symbol.

For example, with the stamp game, all quantities 1,000’s, 100’s, 10’s and 1’s are the same sized small tiles color-coded to the numerals of the bank game. No longer, as with the golden bead materials, is the 1000 cube physically ten times larger than the 100 square, etc. The child now has to abstract that the tile that says 1000, which is a different color, but the same size as the tile that says 100, still represents abstractly ten times the amount.

The child is now moving away from the need for true, accurate, physical representation of quantity, to the more abstract symbolization of quantity. The child will also experience this further, in a more abstract fashion, with the small bead frame.

The child will then move on to the use of the dot board (used for addition) in which the entire math operation is done with recording. For the first time, the child has no concrete materials to manipulate, but must learn to record the problem as well as carry out the process, including exchanging within categories.

Memorization

The child is now ready to move into further “recording” of mathematical processes and also into the initial stage of memorization of math facts. The child is now using abstract materials which still give him or her the opportunity to manipulate the materials on a concrete level, but the child is now becoming less reliant on the materials, and is more able to research, record and remember mathematical information.

In this area of the curriculum, the child continues to work with the four operations. The child also works with the “tables” or” facts” of the operations. The child may now begin to recall or mentally solve mathematical problems. The materials now become an aid or check for the child’s developing ability to retain mathematical information and mentally represent the processes.

Mathematics Reading List

Readings and References:

Montessori, Maria; Dr. Montessori’s Own Handbook

pages 164– 182

Montessori, Maria; The Montessori Method

chapters 12, 13, 14, 19

Montessori, Maria; The Discovery of the Child

chapters 19 & 20

Other Resources:

Montessori Math, by Tim Seldon Tomorrow’s Child; Winter, 2001-02

Montessori Math Moves from the Concrete to the Abstract

Tomorrow’s Child; Fall, 1999

Miscellaneous Resources:

<http://www-groups.dcs.st-and.ac.uk/~history>– I found this to be an easily navigable site with an excellent chronology section. The downside is that it is a UK site and only lists women mathematicians from Britain and Ireland from before 1940, but not Italy!

### Mathematics Concepts for Early Childhood

**Classifying:** Sorting or forming groups by similar attributes Discovering concepts of likeness and differences

Sorting by one attribute (sorting by shape, for example)

Discovering other similarities and regrouping accordingly (sort by color then by shape) Classifying on the basis of negation (squares and non-squares)

Terms: sets, groups, sort, alike, different, words describing shapes, colors, sizes, textures

**Comparing:** Establishing a relationship between objects Comparing amounts or sizes (amounts of milk in a glass, heights, etc.) Comparing numbers of things (determining which set has more)

Comparing sets requiring one-to-one correspondence (one black chip for one red chip)

Terms: more than, greater than, less than, shorter, longer, fewer, least, heavier, equal, same

**Ordering:** Arranging in a sequence

Ordering by size (long to short, thin to thick/fat, large to small) Ordering by number (most to least, least to most)

Ordering by time (first to last, morning, afternoon, night)

Terms: first, second, third, longer, shorter, fewer, fewest, order, sequence, row, line, stack, next, then, later

**Patterning:** A form of ordering containing an element of repetition, should move from simple to complex Becoming aware of patterns (recognizing that the strips of a shirt are in a pattern)

Describing pattern (telling what the pattern looks like)

Extending patterns (changing a red, green, red, green pattern to red, green, blue…) Completing patterns (asking a child to finish a pattern already started – red, blue, red…) Repeating patterns (given a pattern, reproduce or repeat it)

Creating patterns (making up a pattern)

Terms: patterns, alike, different, over and over, repeat, design

**Measuring:** Deciding how long or how much – length, weight, volume Continuous measurement with direct comparisons (placing objects side by side)

Continuous measurement with indirect comparisons (using a stick or string to compare lengths)

Terms: long, longer, shorter, heavier, lighter, more, less, little, big, higher, larger, smaller

**Shape and Space:** Space refers to boundaries, arrangements, and positions; shape refers to form Positions (over, under, above, below, between)

Distance (near, far, close)

Construction (making and changing space, fitting into a space)

Topographical space experiences (altering shapes while retaining properties of being open or closed) Euclidean shape experiences (squares, triangles and other rigid shapes)

**Numbers:**

Experiencing cardinal numbers (how many) Experiencing ordinal numbers (first, second, etc.) Experiencing numbers that are labels (Room 10) Terms: one, two, three, first, second, how many

**Counting:**

Counting by rote (reciting the names of the numerals in order) Counting rationally (attaching a numeral name to a series of objects) Terms: one, two, three, etc.

**Numerals:** Symbols for numbers which should be introduced following an understanding of the cardinality of a set (children can identify and understand how many are in a set)

**Organizing, Representing, and Recording Mathematical Information:**

Drawing, using language to describe, building models, creating simple graphs with real objects

**Problem Solving:** Relating mathematics to the real world and including the following: Real objects

Action or manipulation Interest and challenge

A realization on the part of the child that he has the ability to solve the problem

### Montessori Mathematics Definitions

**Mathematics:** The study of quantities and relationships.

Major branches of mathematics include arithmetic, geometry, algebra and calculus

**Arithmetic:** The science and theory of numbers, and basic number computations through the four

 operations of addition, subtraction, multiplication and division

**Number:** Numbers are ideas, not objects or symbols. Numbers indicate relative quantities. **Numeral:** A symbol that represents a number. Our numerals come from Hindu Arabic signs. **Concrete:** Real objects that can be touched and moved.

**Abstract:** A representation of something real.

**Cardinal Number:** A number which indicates quantity, but does not indicate order. **Ordinal Number:** The order of the series of natural numbers. First, second, third…etc. **Set:** A group of objects.

**The Empty Set:** A set with zero objects.

**Zero:** The number which can be added to any number and yield a sum which is that number.

One of the most important inventions in arithmetic, zero has an important function as a place-holder.

**Decimal System:** Our standard numeration system which uses numerals 0 – 9 and place-value based on

powers of ten.

**Positional Notation:** Recoding numbers by assigning place-value to numerals.

**Operation:** An action to combine or to separate number quantities, i.e., to add, subtract,

multiply or divide.

**Static Operation:** No exchange is necessary.

**Dynamic Operation:** Requires exchanging.

**Basic Addition Facts:** The sums of all pairs of one-digit numbers.

**Basic Subtraction Facts:** The differences between any two numbers when the minuend is no more than 18

and the subtrahend is no more than 9.

**Basic Multiplication Facts:** All possible produces of any two-digit numbers.

**Basic Division Facts:** All possible quotients when the divisor is a number from 1 to 9 and the quotient is a

number from 1 to 9 with no remainder.

**Addition terms:**

Addend: The numbers being combined or added together.

Sum: The answer or resultant quantity when combining different addends

**Subtraction Terms:**

Minuend: The initial quantity

Subtrahend: The quantity being removed or subtracted

Difference: The answer, or amount left after a quantity has been subtracted

**Multiplication Terms:**

Multiplicand: The initial quantity to be “taken”

Multiplier: The number of times a quantity is “to be taken” (or added together)

Product: The answer or resultant quantity once a number has been “take” a number of times

**Division Terms:**

Dividend: The original number to be divided

Divisor: The number of parts the dividend is to be divided into

Quotient: The answer or amount of the divide parts (which will all be equal in amount)

MATH LESSON PLANS

Numeration ~

Number Rods

Sandpaper Numerals Number Rods and Cards Spindle Boxes

Memory Game

Numerals and Counters (Odds and Evens) Mystery Number

Sets Basket Bead Stair

Linear Counting ~

Teen Board Ten Board

Squaring and Cubing Chains Hundred Board

Number Roll

Decimal System Introduction ~

 Introduction to the Decimal System

Introduction to the Decimal System Cards

Association of Decimal Beads to Decimal Cards Large Card Layout

45 Layout

Formation of Complex Numbers

Tray of Equivalence Tray of 9’s

Mystery Quantity Exchange Tray Exchange Game

Decimal System Operations ~

 Static and Dynamic Addition

Static and Dynamic Multiplication Static and Dynamic Subtraction

Static and Dynamic Division

Memorization ~

Snake Game for the Research of 10 Positive Snake Game

Addition Strip Board

Negative Snake Game

Negative Strip Board Multiplication with Bead Bars Multiplication Bead Board

Unit Division Board

Finger Charts

Addition Multiplication Subtraction Division

Passage to Abstraction ~ Stamp Game

 Dot Game

 Small Bead Frame Large Bead Frame

Other Areas of Study ~ Fractions

Geometry

Please choose eight (8) materials for reflective journal entries. Possible material choices are underlined.

# Numeration

## Number Rods

## Sandpaper Numerals

## Number Rods and Cards

## Spindle Boxes

Memory Game

Numerals and Counters (Odds and Evens)

Mystery Number

Sets Basket

## Bead Stair

**Name of Activity: Number Rods**

**Area:** General: Mathematics Specific: Numeration

**Materials:** Ten wooden rods in graded length, one decimeter to one meter, like the length rods, but colored red and blue in alternate decimeters. The number of segments in each rod represents the number of that rod.

|  |  |  |
| --- | --- | --- |
| **Aims:** | Direct: | To develop the Mathematical Mind, to create awareness |
|  |  | and appreciation for the beauty of our number system. |
|  | Indirect: | To learn the quantities of numbers 1-10, to learn the names |
|  |  | of numbers 1-10, to teach that each number is a separate |
|  |  | entity and to count in one to one correspondence on a fixed set. |

**Preparation:** Sorting activities, sensorial activities, especially exposure to the length rods **Age:** First year child

**Presentation of the Lesson:**

1. Bring the child to the shelf where the rods are stored and name the material. Show the child how to carry the rods lengthwise one by one to the mat, starting with the longest rod.
2. Place the rods randomly on the mat with the left end of the rod aligned along the edge of the mat.
3. Build the stair from 1-10, silently, inviting the child to help, when able.
4. Pull the first 3 rods down and give a 3 period lesson on rods 1, 2 and 3. In order, touch each rod and identify it, then count its sections.

“This is one. 1.” “This is two. 1, 2.” “This is three. 1, 2, 3.” (Touch the corresponding colored sections as you say the numbers.)

1. Depending on the interest of the child, give another 3 period lesson on rods 4, 5 and 6. Introduce the 7 through 10 rods in the same manner.
2. Show how to return the rods to the shelf. Start with the longest rod and aligning the red end of the rod along the left edge of the shelf.

**Game #1:**

1. Lay out 2 mats across the room from each other.
2. Place the number rods randomly on one of the mats with left edge aligned to edge of mat.
3. Bring the child to the second mat and ask them to wait there.
4. Bring back a number rod that is larger than 1 and count it. Ask the child to bring a rod that is just one smaller or just one larger.
5. Continue building the stair until it is complete.

**Game #2:**

The game is similar to Game #1, although now the child is asked to bring back a rod that is two smaller or two larger. (The result will be either a smaller stair of either all odd rods or all even rods.)

**Work of the Teacher:**

Points of Emphasis: Careful and safe handling of the material. Counting

the rods precisely and teaching the quantities.

Language: Numbers 1-10, smaller than/larger than

**Points of Interest:** The attractiveness of the material, building a stair and learning to count.

**Work of the Child:**

Points of Consciousness: Numbers have names. The 10 rod is much longer than

the 1 rod.

Control of Error: Visual disharmony of the rods not being aligned.

**Variations:** Child builds rods in sequence at a distance See Games 1 & 2 listed in the presentation.

**Extensions:** Have child make a set of number rods with red and blue paper.

**Name of Activity: Sandpaper Numerals**

**Area:** General: Mathematics Specific: Numeration

**Materials:** A set of symbols from 1-9 cut out of sandpaper, each mounted on a separate wooden card. There is also a 0 card which is not presented at this stage.

|  |  |  |
| --- | --- | --- |
| **Aims:** | Direct: | To develop the Mathematical Mind through hand - mind |
|  |  | activity. To appreciate the genius of humankind in creating |
|  |  | a symbol system to represent numbers. |
|  | Indirect: | Indirect preparation for writing numerals. Indirect preparation |
|  |  | for decimal system, linear counting and abstraction. |

**Preparation:** Preparatory exercises for the discrimination of touch; touch boards, fabric boxes and number rods.

**Age:** First year child **Presentation of the Lesson:**

1. The child can sensitize their fingertips by rubbing them briskly on the rug or a rough fabric.
2. Say to the child, ”I’m going to choose 3 special numbers.” Teacher takes the numerals 1, 2, 3 and places them face down on the table but in order of presentation.
3. Turn over the first card. Say “1”. Steady the card with one hand and trace the numeral and say “One”. Invite the child to trace the card.
4. Place the card in the upper left of the mat, face up. Place traced cards face up from left to right.
5. Repeat with the other cards. Proceed with a 3 period lesson for the numerals 1, 2 & 3.
6. May repeat with other sets of numerals, depending on child.

**Work of the Teacher:**

Points of Emphasis: Sensitizing the fingertips, proper tracing of the

numerals and giving the language.

Language: Numbers 1-9

**Points of Interest:** Tactile experience of tracing the numerals. **Work of the Child:**

Points of Consciousness: Numerals are written symbols representing

quantities.

Control of Error: Outline of the sandpaper numeral.

**Variations:** Play games with the sandpaper numerals once the child is at the third period with the lesson.

**Extensions:**

**Name of Activity: Number Rods and Cards**

**Area:** General: Mathematics Specific: Numeration

**Materials:** The number rods. A set of wooden cards on which are printed the numbers 1-10.

|  |  |  |
| --- | --- | --- |
| **Aims:** | Direct: | To develop the Mathematical Mind, awareness and |
|  |  | appreciation for genius of our system of numbers. |
|  | Indirect: | To associate quantity with a symbol. To reinforce the |
|  |  | correct sequence of numbers 1-10. To introduce the |
|  |  | processes of addition and subtraction. |

**Preparation:** Number rods, sandpaper numerals **Age:** First year child

**Presentation of the Lesson: Preliminary Presentation:**

1. This can be done as a matching game.
2. Have the child build the rods in order from 1-10.
3. Take the number cards out of the box one by one and place in order across the top of the rug.
4. Place your hand on the 1 number rod and say, “This is one.” Hold up the number card 1 and say, “This says one.”
5. Place it against the 1 rod. Continue by counting number rod 2 and say, “This is 2.” Place card 2, saying, “This says two.”
6. Repeat with each number rod and card in order up to 10.
7. Children have not seen the number 10, so, pause when you take the number card 10 out and show it to the child. “I don’t think you have seen this number before. This is 10, a 1 with a zero says 10.”

Note: Sliding the number rods down from the stair before each counting allows for greater isolation of the quantity being counted and the displaying of the card.

**Presentation #2:**

1. Invite the child to the lesson and name the activity.
2. Bring the rods to the work mat and lay them out across the mat in random order.
3. Invite the child to build the number rods in order.
4. Introduce the card material and place in random order across the top of the mat.
5. Touch the number 1 rod and count “1” (“This is one”). Deliberately search for the number card for 1 and place against the end of the number rod (“This says one.”).
6. Repeat for 2. Invite the child to take over when they appear ready.
7. At the end of the exercise, point to each numeral and say that number to reinforce the symbol, counting 1-10. May repeat activity with child.
8. Replace the number cards to their box, placing them in order with one in front. Return number rods on shelf, starting with the longest rod.

**Presentation #3:**

1. Invite the child to the lesson and tell them that, “We are going to do something different today.”
2. Lay out the number rods in random order as before.
3. On a separate mat, lay out the number cards.
4. Choose a number card and say the number. Ask the child to bring the number rod to match the card.
5. When the child returns with the rod count the rod and place the number card at the end of the rod.
6. Continue the work as above.

**Presentation #4:**

1. Lay out a work mat. Bring the number cards and place in random order.
2. Lay out a second work mat at a distance and lay out number rods in random order.
3. This time, ask the child to count a number rod and fetch the corresponding number card. Count the rod to confirm and place the card at the end of the rod.

**Presentation #5:**

1. Lay out a work mat and place the number rods in random order.
2. Lay out a second mat with the number cards laid out randomly.
3. Show the child one of the number cards and ask the child to bring over the number rod which is one smaller/less than the number shown.
4. Child leaves card on the mat while they get the correct rod.
5. Count the rod to confirm its value and place corresponding number card.
6. Repeat step 3, asking for one larger/greater.

**Presentation #6:**

1. Lay out materials as in Presentation #4.
2. This time, ask the child to count a number rod and bring the number card that is one less than the rod.
3. Count the rod to confirm and place corresponding number card.
4. Repeat step 2, asking for one larger/greater.

**Presentation #7:**

1. Place the number rods on one mat in random order. Lay out a second mat at a distance and bring the number cards.
2. Choose a number and show to the child. Ask them to bring the rod that is 2 smaller or larger than the number shown.

**Presentation #8: Making 10’s: Introduction to Addition**

1. Bring the number rods to the mat and place in random order.
2. Invite child to build the stair, beginning with the **1** number rod.
3. Arrange number cards along top of mat in order from left to right.
4. Isolate the rod of **10** by moving towards the top of the mat.
5. Count the **10** rod and place the number card for 10 at the end of the rod.

(Note: As you begin to move up additional rods to isolate, be sure to leave space between previous rods for placement of the cards.)

1. Bring up the **9** rod and count. “How many more do we need to make 10?”
2. Count the **1** rod and move up and place beside the **9** rod. Count to 10.
3. Place number cards for 9 and 1. Say, “9 and 1 together make 10.”
4. Bring up the 8 rod and count the rod. “How many more do we need to make 10?”
5. Count the **2** rod and bring up and place beside the **8** rod. Count to 10.
6. Place the number cards for 8 and 2. Say, “8 and 2 together make 10.”
7. Continue in this manner until the rod **5** is reached. Count to 5. “How many more do we need to make 10?”
8. “I’m going to show you something special. Count the rod of **5** and then flip rod from left end and count again, continuing from 5 (6, 7, 8, 9, 10).
9. Place number card for 5 at end of rod after flipping. “5 and 5 make 10.”
10. “Let’s count how many 10’s we have made.” Count four 10’s. Say to the child that “We have made four 10’s with 5 left over (because there is only one rod of 5).

\*Note: Each time 10 is made, be sure to count to ten, touching the colored sections of the rods.

Continue straight into subtraction:

1. Say to the child, “If we have 10 and take away 5, how many do we have left?”
2. Demonstrate and count. (The 5 rod can be used to verify remaining amount, if needed, by reflipping back to its original placement.) “If we have 10 and remove 5, we have 5 left.”
3. Continue in this manner by removing the 4 rod (place beneath 5 rod).
4. Ask, “If we have 10 and take away 4, how many do we have left? Count remaining 6 rod. (“If we have 10 and take away 4, we have 6 left.”)
5. Continue with other rods. (As rods are removed, place with other removed rods in stair. Cards may be placed randomly at bottom of mat.)
6. Help child to put away materials in proper order.

**Work of the Teacher:**

Points of Emphasis: Introduce the symbol card for 10. Careful and thorough

presentations for addition and subtraction.

Language: Numbers 1-10/ smaller/less than/greater than, addition, subtraction.

**Points of Interest:** Discovering new ways to use the number rods. Playing games with the number rods and cards.

**Work of the Child:**

Points of Consciousness: Each number rod has a corresponding written

symbol.

Control of Error: Stair built correctly

**Variations:** See presentations

**Extensions:** The concept of greater than and less than, without symbols.

**Name of Activity:** **Spindle Boxes**

**Area:** General: Mathematics Specific: Numeration

**Materials:** Two open boxes divided into five compartments each or one open box divided into ten compartments. Each compartment is numbered 0-9 in sequence.

A basket containing 45 spindles.

**Aims:** Direct: To develop logic, order and reasoning through hand-mind

activity. To further develop an appreciation of the number system.

Indirect: To help children recognize a symbol and its corresponding quantity. To introduce zero as an empty set.

**Preparation:** Number rods, sandpaper numerals and the association of the numerals and counting rods

Note: In this activity, the child now must create specific ‘sets’ of individual objects. Previously, the number rods presented each ‘set’ or quantity as a complete and fixed entity.

**Age:** First year child **Presentation of the Lesson:**

1. Invite the child to the lesson and name the material at the shelf. Carry the materials individually to the work place and set the boxes beside each other with the basket of spindles to the left.
2. This is a good opportunity to review the numerals and to determine if the child has mastered the recognition of the numerals 0 through 9.
3. Point to the numeral 1 and say; “this says 1.”
4. Place one spindle into the child’s hand and say, “This is 1.”
5. Point to the numeral 2 and say, “This says 2.” Count and place 2 spindles into the child’s hand and say, “This is 2.”
6. Continue with the remaining numerals. Allow the child to take over the process at any point.
7. Once all the spindles are counted into their compartments, pause to see if the child comments about the zero compartment.
8. If the child does not comment, say, “Zero is an empty set. It is a place holder.” It is important to not give the child the impression that zero is “nothing.”
9. Show the child how to tidy up the work and replace it on the shelf.

\*Note: There are just enough spindles to complete the work exactly.

Watch for inaccuracies. If this occurs, say to the child, “Perhaps we can count the

 set again.” **Work of the Teacher:**

Points of Emphasis: Counting the spindles directly into the child’s hand.

Introducing zero as a place holder.

Language: Numerals 1-9, zero, placeholder, set, empty set.

**Points of Interest:** The zero compartment is empty. **Work of the Child:**

Points of Consciousness: The spindle bundles grow as the number increases.

Control of Error: A set of 45 spindles is sufficient to complete the activity correctly.

**Variations:**

**Extensions:** Create “sets’ by binding the bunches of spindles. (Use green ribbons or elastics to coincide with the color green for units.) Games to reinforce the concept of zero.

**Name of Activity:** Memory Game

**Area:** General: Mathematics Specific: Numeration

**Materials:** Tickets with numbers 0-10, 55 objects in a container, a tray.

**Aims:** Direct: To develop the Mathematical Mind. To create awareness

of numbers in the world.

Indirect: To count 1-10. To recall a number and find a representative quantity. To reinforce the concept of 0 as an empty set.

**Preparation:** Numeration work, including the spindle boxes. **Age:** First year

**Presentation of the Lesson:** Presentation #1:

1. Invite a small group of children to play this game.
2. Give a command to each child in turn to perform an action. For example: “clap 7 times”, “tap your shoulder 9 times”, etc.
3. Zero is not introduced in the first presentation but is introduced in a later game.

Presentation #2:

1. Gather a group of children. Place the basket on a table or rug in another section of the room.
2. Write a number ticket and hand it to a child. Ask the child to read the number and place it face down on the mat. “Don’t let anyone see your number.”
3. Child fetches objects from the basket and brings them back to the mat. Child counts the objects. “Can anyone guess what (child’s) number was?”
4. Turn the slip over and ask child to read their number.
5. Continue with the game.

**Work of the Teacher:**

Points of Emphasis: Clarity in explaining instructions around the game.

Writing numeral clearly.

Language: Mystery, 0-10, names of objects

**Points of Interest:** Playing a game, counting, remembering numbers and finding matching quantities from memory.

**Work of the Child:**

Points of Consciousness: Zero is an empty set. Remembering the number.

Math is fun!

Control of Error: Number on ticket

**Variations:** Vary objects used with the game. (Can reinforce ‘themes’ being studied within the class.)

**Extensions:** Use objects from the environment (nature, also).

**Name of Activity:** **Numerals and Counters (Odds and Evens)**

**Area:** General: Mathematics Specific: Numeration

**Materials:** Cut out wooden numerals 1-10, 55 counters of one color, container.

**Aims:** Direct: Further development of the Mathematical Mind. To develop

logic, order and reasoning.

Indirect: To offer further practice in counting. To sequence numerals in correct order. To introduce the concept of odd and even.

**Preparation:** All previous numeration work, much counting practice. **Age:** First to second year child

**Presentation of the Lesson:**

1. Invite child to take out the numerals and place randomly on the mat.
2. Place the 1 card in the top left corner and show the child how to make a space between numerals (child may use width of own hand).
3. Have child lay out other numeral cards 1-10 from left to right.
4. Teacher starts to count counters. Place 1 counter under the numeral 1.
5. Count 2 counters, one at a time, placing 1 and 2 side by side under the numeral 2 with slight space between counters.
6. Count 3 counters one at a time, placing 1 and 2 side by side and the third counter in the middle under the first row.
7. Continue in this manner up to 10.
8. Place finger at center of each row and move it gently top to bottom saying, “odd”, “even”, “odd”, “even”. On even rows finger will slide between the counters, on odd rows finger will stop at “odd” counter at bottom of row.
9. Say to the child, “Some numbers are odd, some numbers are even.”
10. Invite the child to place the counters under the numerals again.
11. Replace work carefully.

**Work of the Teacher:**

Points of Emphasis: The placement of the counters. Showing the child

how to differentiate between odd and even numbers.

Language: Odd and even, reinforcement of numbers 1-10

**Points of Interest:** Attractiveness of materials and color **Work of the Child:**

Points of Consciousness: That numbers can be even or odd, quantity increases

as numerical value increases.

Control of Error: There are 55 counters sufficient to complete the exercise.

**Variations:** Vary the objects used in the game.

**Extensions:** Making odd and even booklets.

**Name of Activity: Mystery Number**

**Area:** General: Mathematics Specific: Numeration

**Materials:** Three containers, one for holding objects to be counted, one with blank strips of paper, one for holding slips of paper with recorded mystery number, tray.

**Aims:** Direct: To develop the Mathematical Mind.

Indirect: To practice counting.

**Preparation:** Numeration activities, fingerplays, songs and stories for counting practice. **Age:** First year child

**Presentation of the Lesson:**

1. Dramatize opening the container and removing the objects one by one.
2. Quietly count the objects and say the number.
3. Take a slip of paper and write your name and the number of objects counted.
4. Place the slip of paper into the answer box. Replace the objects and invite the child to have a turn.
5. Tell the child they can choose this activity everyday.
6. At Circle, count the objects with the children and read the slips. If someone has written a different number, acknowledge the effort. For example, you might say, “Sarah did the mystery numbers today” or “Sam loves to count.”

**Work of the Teacher:**

Points of Emphasis: Counting each object individually. Model writing the

numeral.

Language: numbers and objects

**Points of Interest:** Discovering what is inside the box, counting the objects, getting slips back at Circle.

**Work of the Child:**

Points of Consciousness: Applying counting principles to unknown situations. Control of Error: No fixed control.

**Variations:** Vary the objects. Relate the objects to unit themes or curriculum areas.

**Extensions:**

**Name of Activity: Sets Basket**

**Area:** General: Mathematics Specific: Numeration

**Materials:** Work mat, collection of 55 objects for numbers 0-10, set of number tags.

**Aims:** Direct: To develop the Mathematical Mind, to develop logic, order

and reasoning, to create awareness and appreciation for beauty our number system.

Indirect: To count 0-10, to reinforce zero as an empty set.

**Preparation:** All previous numeration work. **Age:** First to second year children. **Presentation of the Lesson:**

1. Bring the work to a mat.
2. Lay the number tags across the top of the mat in random order.
3. Say to the child, “We are going to take all the sets out of the basket and put together the ones that are the same.”
4. Take the objects out of the basket and lay on the mat.
5. Group the like objects together (put into sets).
6. Say to the child, ”I’m going to choose a set of objects to count.”
7. Count a group of objects and match to the number tag. Carefully search for the number and place the objects beneath the number tag.
8. Invite the child to choose a set to count.

**Work of the Teacher:**

Points of Emphasis: Laying out the number tags, grouping objects into

sets, counting carefully.

Language: Objects in the activity, numbers, zero as an empty set.

**Points of Interest:** The beauty of the objects, seeing the objects change.

**Work of the Child:**

Points of Consciousness: Zero is an empty set, conservation of number Control of Error: 55 objects is sufficient to complete the exercise

**Variations:** Change the objects often, make baskets thematic or seasonal.

**Extensions:**

**Name of Activity: Bead Stair**

**Area:** General: Mathematics Specific: Numeration

**Materials:** A set of colored bead bars each representing the numbers 1-9, each bar distinguishes each number up to 9 as a separate entity; 1 is red, 2 is green, 3 is salmon/peach, 4 is yellow, 5 is light blue, 6 is light purple (lavender), 7 is white, 8 is brown, 9 is dark blue. A small mat to display the bead stair and a tray.

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| --- | --- | --- |
| **Aims:** | Direct: | To develop the Mathematical Mind and further develop the child’s |
|  |  | intellect through hand - mind activity. |
|  | Indirect: | To prepare the child for future abstract mathematical |
|  |  | activities involving this material. To associate color and quantity |
|  |  | of the bead material. |

**Preparation:** All previous numeration activities. **Age:** Second year child

Presentation of the Lesson:

1. Take the bead stair to the work mat or table. Take out the bead bars and place randomly on the table.
2. “Let’s find 1.”
3. Find the 1 bar (red), count and place at the top center of the mat.
4. Search carefully for 2, count each bead and place into the stair below the 1 bar.
5. Repeat finding and counting the bead bars until the stair is complete.
6. Mix the bead bars and rebuild the stair.

**Work of the Teacher:**

Points of Emphasis: Searching for each quantity, touching each bead

separately as you count. Building the stair.

Language: numbers and colors

**Points of Interest:** Beauty and “feel” of the material, learning that each color represents a different quantity, building a stair.

**Work of the Child:**

Points of Consciousness: Each colored bar represents a different quantity. Control of Error: Child’s own knowledge of counting numbers 1-9.

Visual disharmony in the stair.

**Variations:** Play memory games to reinforce color and quantity.

**Extensions:** Add paper for reinforcement of color recognition.

Linear Counting

Teen Boards Tens Boards

Squaring and Cubing Chains

100 Board

Number Roll

**Name of Activity: Teen Beads and Boards**

**Area:** General: Mathematics Specific: Linear Counting

**Materials:** Two wooden boards printed with 9 ten’s, a set of cards which slide into the boards covering the zero. These cards are printed with the numerals 1-9. A box containing 9 golden ten bars and a colored bead stair, a felt mat for the beads.

|  |  |  |
| --- | --- | --- |
| **Aims:** | Direct: | To further develop the Mathematical Mind and to increase the |
|  |  | child’s awareness of both the simplicity and complexity of our |
|  |  | numerical system. |
|  | Indirect: | To learn the quantity for the teen numbers and to learn the |
|  |  | names of the teens and introduce concept of place value. |

**Preparation:** All previous numeration work. This work can run parallel to the decimal system. **Age:** Second year

**Presentation of the Lesson:**

**Presentation #1:** Introduction to the Teen Beads (without Boards)

1. Bring the box of beads to the work space and ask the child to build the bead stair on the right of the rug.
2. Line up the ten bars below the bead stair (place bars vertically).
3. Place a ten bar at the top left of the rug. Place the red one bar to the right of the ten bar and say, “10 and 1 is 11”. Count the beads 1-11. (Count down the ten bar and back up for the one - count slowly.)
4. Make 12 beneath 11. Count the beads for 12. Continue in this manner forming and counting the beads for 13. Follow with a 3 period lesson on 11, 12 and 13.
5. Depending on the child proceed to form numbers 14, 15 and 16, followed by a 3 period lesson.
6. Continue in this manner with 17, 18 and 19.

**Presentation #2:** Teen Boards (without Beads)

1. Lay the teen boards on the mat, top to bottom.
2. Place the number cards on the mat (randomly) and then make a stack face-up with one on the bottom.
3. Turn the pile of number cards face down on the right side of the rug.
4. Point to each 10 on the board and say, “This says 10.”
5. Turn over the first number card and slide it over the zero of the first ten on the board and say, “This says 11.”
6. Continue in this manner, forming numbers 12 and 13. Follow with a 3 period lesson on 11, 12 and 13.
7. Depending on the child, proceed to form the numbers 14, 15 and 16, followed by a 3 period lesson.
8. Continue in this manner with the numbers 17, 18 and 19.

**Presentation #3:** Association of Quantities with Symbols (Beads and Boards together)

1. Bring the teen boards and the box of beads to the work rug.
2. Lay out the boards and organize the number cards into a pile as before.
3. Organize the beads on the felt mat in the lower right corner (build bead stair and line up the 10 bars vertically beneath the stair).
4. Place a ten bar and a one bar to the left of the top section. Say, “This is 11.” Count the beads.
5. Slide the number 1 card into the top slot of the board covering the zero and say, “This says 11.”
6. Continue placing and counting beads and sliding corresponding numeral cards onto board for numbers 12-19.

Note: The beads are placed to the left of the boards so as to not interfere with the sliding of the number cards when used together.

**Work of the Teacher:**

Points of Emphasis: Using the language “this is” (for quantities) and

“this says” for symbols. Placement of beads and care sliding of cards.

Language: The names of the teen numbers.

**Points of Interest:** Working with the combination of beads and quantities.

**Work of the Child:**

Points of Consciousness: Quantities can be represented by symbols. Control of Error: Child’s knowledge of numerals 1-9, the teacher.

**Variations:** Form the number on the board and have the child find the quantity.

Mix up the number cards randomly and have the child place the numbers in correct sequence.

Mix up the quantities and have the child match the correct number cards to the beads.

**Extensions:** Writing the teen numbers, teen number worksheet.

Source: NMI; February 2003

**Name of Activity: Tens Boards**

**Area:** General: Mathematics Specific: Linear Counting

**Materials:** Two boards similar to the teen boards with numbered slots 10 - 90. A set of cards 1-9. A set of 45 ten bars for presentation 1; 9 ten bars and 10 unit beads for the other presentations. A felt mat for the beads.

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| --- | --- | --- |
| **Aims:** | Direct: | To further the child’s awareness of the simplicity and |
|  |  | complexity of our numerical system. To further develop |
|  |  | the child’s sense of logic, order and concentration. |
|  | Indirect: | To continue linear counting, to learn to count numbers |
|  |  | from 10-90 and 11-99. |

**Preparation:** Teen boards with beads, the hundred board and decimal system. **Age:** Second year

**Presentation of the Lesson:**

**Presentation #1:** Introduction to the Boards and Ten Bars

1. Layout the boards, one below the other.
2. Layout the felt mat in the lower right corner and place the ten bars in a row vertically on the mat.
3. Place a ten bar beside the 10 slot and say, “This is ten.” Point to the printed 10 and say, “This says ten.” Count the 10 bar if the child needs reinforcement.
4. Place 2 ten bars to the left of the 20 slot and repeat the above. (First, refer to the bars, “this is 20” and then the board, “this says 20”.
5. Continue placing ten bars and associating them to the number symbol up to 90.

**Presentation #2:** Boards with Number Cards and Beads

1. Bring the materials to the mat and organize in an orderly way, placing the boards in the center of the mat and the beads in the lower right corner.
2. Organize the number cards in sequence and place in a pile face up in the upper right of the mat.
3. Place a ten bar and a unit bead to the left of the first slot. Count the beads and say, “This is 11.” Slide the 1 card into the slot as you say, “This says 11.”
4. Add a unit to the beads and count to 12. Remove the 1 card and place face down in a discard pile. Slide in the 2 card and say, “This says 12.”
5. Continue placing beads, counting and associating the symbols up to 19.
6. Say to the child, “If we add one more unit, we will have 10.” Add one more unit, then remove the 10 units and replace them with a 10 bar.
7. Move both ten bars down to the 20 slot and turn over the number pile.
8. Repeat the above process. The procedure is the same for all remaining numbers.

**Presentation #3:** Forming Random Numbers

1. Lay out the number cards randomly on the mat and the ten boards in the center of the mat. Arrange the beads on the felt mat.
2. Ask the child to make a number with the cards. For example, say to the child, “I’m thinking of the number 58.”
3. The child forms the number on the board and places the appropriate beads beside the number.
4. This activity can be done by forming numbers only. Additional bead material will need to be borrowed from the bank.

**Work of the Teacher:**

Points of Emphasis: Layout of the materials. Clearly demonstrating the

transition from 19 to 20, 29 to 30, etc.

Language: 10-90 and 11-99

**Points of Interest:** Working with large numbers, making exchanges and working with the beads and the boards.

**Work of the Child:**

Points of Consciousness: Numbers 1-9 remain fixed but indicate a higher

quantity when in a different place. The hierarchy of a number changes after 9 is reached.

Control of Error: The child’s knowledge of numbers, the teacher or a

control chart.

**Variations:** Mixing numbers randomly, making quantities with the beads and having child make the proper number.

**Extensions:** Writing the numbers.

Source: NMI; February 2003

**Name of Activity: Squaring and Cubing Chains**

**Area:** General: Mathematics Specific: Linear Counting

**Materials:** The golden 100 chain, 10 ten bars, a box of labels containing 1-9 in green, tens in blue, 100 in red, a golden 100 square.

The golden 1000 chain of 100 ten bars, a box containing labels 1-1000, 10 golden hundred squares and 1 golden thousand cube.

**Aims:** Direct: Further development of the Mathematical Mind, an

appreciation of the number system.

Indirect: To learn to skip count, to learn the square and cube values of numbers 1-10, preparation for the multiplication tables.

**Preparation:** In conjunction with the 10 board and 100 board. **Age:** Second to Third year

**Presentation of the Lesson:**

**Presentation #1:** 10 Squaring Chain (no labels)

1. Invite the child to the activity and show the child how to carry the chain and square from the cabinet to the rug.
2. Place the square to the left side of the work mat.
3. Arrange the chain under the square and stretch it out to its full length.
4. Show the child how to fold the chain in an accordion like manner to make a square. The square formed by the chain will still be under the fixed square.
5. Show the child that the squares are the same by fitting the fixed square over the folder square.
6. Secure the chain at one end and slowly draw the chain out to its full length again.
7. Say to the child, “I wonder how many beads are in this chain?”
8. Count 1 by 1 each bead from 1-100. (May ‘pinch’ beads while counting.)

**Presentation #2:** 10 Squaring Chain (with labels)

1. Lay out the materials as in Presentation #1.
2. Remove the number tags and arrange in the lower right area of the mat.
3. If a review is necessary, fold the chain to show that it is the same as the fixed square.
4. Start to count the beads one by one. Invite the child to join the activity when they understand the process.
5. When you reach the end of the first bar, place the first tag (10) at the last bead on the first bar of the chain.
6. When all the number tags have been placed, read the tags in order.
7. When the materials are returned to the cabinet, invite the child to count any of the square chains.

**Presentation #3:** Flipping Number Tags

1. Proceed as in Presentations 1 & 2.

2. Read all of the number tags: i.e., 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.

1. Go back to the first number tag and turn it over. Say 10.
2. Continue to count the remaining number tags.
3. Go back to the beginning and flip over the 20 tag. Start to count again from the beginning of the chain 10, 20, 30...
4. Repeat process until all the number tags have been flipped over, each time counting again from the beginning.

Note: If the child is able to recite the numbers accurately in sequence, you can say, “You said all of the numbers, but we can’t see them. You are holding the numbers in you head!” (\*NOTE: All of the multiples of numbers for memorization are represented within the square tickets.)

**Presentation #4:** Cubing Chain (10 cube chain or 1000 chain)

The cubing chain is presented to the child in the same manner as the 100 chain with the exception that the labels are included from the beginning.

1. The cube is brought to the mat and laid carefully on the top left corner of the mat.
2. Bring the ten 100 squares to the mat and lay them out left to right along the top of the mat.
3. The chain is then brought over carefully and laid out. This will require a series of mats or a strip that has been especially prepared for this work.
4. Fold the chain into 10 squares and place a fixed square beside each folded square.
5. Show the child that the cube is made up of 10 squares. Count the number of squares laid out, and then count the number of squares in the cube. Carefully build the 100 squares into a (loose) cube. Compare this to the ‘fixed’ cube.
6. Stretch the 1,000 chain out to its full length and replace the ten 100 squares at the end of each section of the chain. Count and label as before with the 100 chain.

Note: When the 10 cube chain is laid out, you may also take the chain of 100 and move it along the 1000 chain to show how it is made out of ten 100 chains joined together.

**Work of the Teacher:**

Points of Emphasis: Careful handling of the material, how to remove and

replace in the cabinet. Folding chain into squares.

Language: Square, cube, numbers

**Points of Interest:** Beauty of the squaring and cubing materials. The length of the chains. Counting high numbers.

**Work of the Child:**

Points of Consciousness: Visual perception of what a ‘square’ and a ‘cube’

are in numerical terms. You can count in number “chunks” (skip counting)

Control of Error: Number tags in order, the teacher.

**Variations:** Place number tags on a separate mat.

After flipping activity, point to tickets randomly and ask child, “How much is this?” Turn over ticket to check.

Repeat until all tickets are right side up again.

Also, may have all tickets turned upside down on rug beside chain. Turn over tickets one at a time, identify number and place next to chain. The check would be the increase of the values of the tickets once all tickets are beside the chain.

**Extensions:** Make a picture of the chain, using a bingo marker and a long strip of paper.

**Notes:** Up to now, the child has worked with teens and tens and has worked with hundreds in the decimal system. With this activity, the child becomes familiar with sequence.

Source: NMI; February 2003

**Name of Activity: 100 Board**

**Area:** General: Mathematics Specific: Linear Counting

**Materials:** 100 board, tiles for numbers 1-100

**Aims:** Direct: To further develop the Mathematical Mind, to develop logic, order

and reasoning, to appreciate the beauty of numbers.

Indirect: To practice linear counting, to sequence numbers from 1 – 100 and to reinforce the symbols for 1-100.

**Preparation:** Numeration activities, teen and ten board sequences, 10 square chain. **Age:** Second Year

**Presentation of the Lesson:**

1. Invite the child to the activity and name the material. Bring the material to the work area.
2. Take out one set of tiles (that correspond to the first row of numbers).
3. Place the numbers randomly on the right. Say to the child, “When we start to count, we always start with 1.”
4. Place the number tiles on the board starting with the 1 tile.
5. Continue with the next group of number tiles.
6. Return tiles to the original box in numerical order 1-50 and 51-100.

**Work of the Teacher:**

Points of Emphasis: Helping the child to organize the tiles. Language: Numbers 1-100

**Points of Interest:** Using the 100 board, placing the number tiles on the board.

**Work of the Child:**

Points of Consciousness: To see the progression of numbers. Control of Error: Control chart

**Variations:** Mix the number tiles up and have the child arrange them in proper order.

With tiles in order on board, ask child to close

eyes. Turn over or remove one tile. Ask child which number is missing.

**Extensions:** Writing the numbers on a control chart. Design a control chart with numbers missing.

**Notes:** There are different ways to handle the containment of the tiles for the 100 board. They can be sorted by rows or in combinations of 2-3 rows.

The hundred board can also be returned to, once children are working with multiples of numbers to investigate patterns which exist in the layout of the numbers.

**Name of Activity:** **Number Roll**

**Area:** General: Mathematics Specific: Linear Counting

**Materials:** Grid paper cut into strips, scotch tape and a pencil.

**Aims:** Direct: Further exposure to the logic, order and reasoning needed

to develop the Mathematical Mind.

Indirect: To practice linear counting and the recognition of numbers.

**Preparation:** Numeration materials, introduction to the decimal system and linear counting materials.

**Age:** Second to third year **Presentation of the Lesson:**

1. The child writes their name at the top of the paper.
2. The teacher explains how each number is written on a separate line in the grid, starting in the top, right corner and working down the sheet.
3. Extra grid sheets are taped on as needed.

**Work of the Teacher:**

Points of Emphasis: Starting in top right hand box and using one line of

grid for every number.

Language: Names of numbers, “exchange”.

**Points of Interest:** Long piece of work! **Work of the Child:**

Points of Consciousness: Once the names and sequences of numerals are

known, high quantities can be counted and written.

Control of Error: Control chart **Variations:** None

**Extensions:** Writing numbers on plain strips of paper. Writing

numbers using green, blue and red to color code categories.

Decimal System Introduction

Introduction to the Decimal System
Introduction to the Decimal System Cards

Association of Decimal Beads to Decimal Cards

Large Card Layout

45 Lay-out
Formation of Complex Numbers

Tray of Equivalence

Tray of 9’s (Exchange Tray)

Mystery Quantity Exchange Tray

Exchange Game

**Name of Activity: Introduction to the Decimal System**

**Area:** General: Mathematics Specific: Decimal System

**Materials:** Tray containing bead material to introduce the four hierarchies (units, tens, hundreds and thousand) one of each.

|  |  |  |
| --- | --- | --- |
| **Aims:** | Direct: | To further develop the Mathematical Mind. |
|  | Indirect: | To introduce the hierarchy of numbers and to present the child |
|  |  | with a concrete, sensorial, experience of the decimal hierarchy. |

**Preparation:** Numeration activities, knowledge of numbers 1-10. **Age:** First year child

**Presentation of the Lesson:** Introduction

This lesson can be done with a small group of children, in which case, the children take turns handling the material.

1. Bring the tray to a work mat and invite the children to sit around.
2. Introduce each category, starting with the unit. “This is one unit.”
3. Let the children handle the unit bead.
4. Place the 10 on the mat. “This is one ten. Let’s count the beads.” Count each unit bead 1-10. (May wish to have child ‘pinch’ each bead between index finger and thumb as they say the number aloud.)
5. Let the children handle the 10.
6. Introduce the 100. “This is one hundred.” Show the child that it is made up of 10 tens. Show the relationship of the 10 to the one hundred and count the tens, 1 ten, 2 tens, 3 tens... up to 10 tens.
7. Place the thousand on the mat. “This is one thousand; It is made up of 10 hundreds.” Show the relationship of the hundred to the thousand by counting the hundreds: 1 hundred, 2 hundred.... to 10 hundreds.
8. Give a 3 period lesson on the beads.

**“Verbal Fetching”**

After introducing the various categories of the decimal system material, show the child the ‘bank game’ supply and have her/him practice “fetching” quantities from the various categories):

First child: “Bring me six units.” Second child: “Bring me five units.”

Repeat with successive categories, having child count quantity and name category each time. Repeat as often as possible. We wish to make this a concrete experience and a firm foundation for later work. This work serves as an extension of the second and third periods.

**Work of the Teacher:**

Points of Emphasis: Keeping the language very specific and giving the

explanation as precisely and minimally as possible so that the child’s focus is clearly on the association of the material with the name.

Language: units, tens, hundreds and thousands

**Points of Interest:** Sensorial experience of size and weight of the different categories of the decimal system.

**Work of the Child:**

Points of Consciousness: units, tens, hundreds and thousands belong to

different categories

Control of Error: the teacher **Variations:**

**Extensions:**

Source: NMI; February 2003

**Name of Activity: Introduction to the Decimal System Cards**

**Area:** General: Mathematics Specific: Decimal System

**Materials:** Set of large Decimal System Cards representing 1 unit, 1 ten, 1 hundred and 1 thousand.

**Aims:** Direct: To develop the Mathematical Mind, to appreciate the beauty of mathematics.

Indirect: To prepare the child to move towards abstraction. To recognize the symbols of the decimal hierarchies.

**Preparation:** Previous numeration work, introduction to the decimal system beads. **Age:** First year

**Presentation of the Lesson:**

1. This lesson can be presented to a small group.
2. Lay out the 4 introductory number cards across the mat, with the unit card at the right, then the ten, then the one hundred and lastly, the one thousand card, sequencing right to left.
3. Point to the unit card and say, “This says 1 unit.”
4. Point to the ten card and say, “This says one ten.” Point out the zero. (“There is one zero to ‘keep the place of the units’.”)
5. Point to the hundred card and say, “This says one hundred.” Point out the zeros. (“There are two zeros to keep the place of the units and of the tens.”)
6. Repeat for the thousand card, being sure to point out the zeros.
7. Give a 3 period lesson. During the lesson point out the zeros, “Please show me the one hundred card. How many zeros are there in one hundred?”

**Work of the Teacher:**

Points of Emphasis: Laying out the cards from right to left, pointing

out the zeros in the numbers.

Language: Numerical values of the hierarchies.

**Points of Interest:** Seeing large number cards and counting the zeros.

**Work of the Child:**

Points of Consciousness: Zero is a place holder. Control of Error: The teacher.

**Variations:** Playing memory games with the cards.

**Extensions:** To associate the cards to the quantities of the golden beads.

Source: NMI; February 2003

**Name of Activity: Association of Decimal Beads to Decimal Cards**

**Area:** General: Mathematics Specific: Decimal System

**Materials:** Introduction tray of decimal beads, box with the symbols for 1, 10, 100 and 1,000.

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| **Aims:** | Direct: | To further develop the child’s understanding of the |
|  |  | decimal system. |
|  | Indirect: | To associate the quantity (beads) to symbols. |

**Preparation:** Introduction tray, introduction to cards, previous numeration work. **Age:** Second year child.

**Presentation of the Lesson:**

1. Bring the materials to the work mat and place the tray of beads in the center of the mat and the number cards to the lower right.
2. Review the golden beads with the children (unit, ten, hundred and thousand).
3. Arrange the beads across the mat with the unit bead on the right, followed by the ten, hundred and thousand.
4. Point to the unit bead and say, “This is one unit.” Place the numeral card beneath the unit saying, “This says 1 unit.”
5. Repeat for the ten, hundred and thousand, using the above language of “this is...” and “this says...”
6. Give a 3 period lesson, mixing up the numeral cards and beads and having child place them in proper combination.

**Work of the Teacher:**

Points of Emphasis: Arrangement of the materials, sequencing materials

from right to left. Precise language.

Language: Language for the decimal system

**Points of Interest:** The beauty of the “golden beads”

**Work of the Child:**

Points of Consciousness: Each quantity has a symbol.

Control of Error: Child’s knowledge and recognition of numerals and quantities. The teacher.

**Variations:** Games played with the materials during the 3 period lesson.

**Extensions:** Preparation for further work with the decimal system.

Source: NMI; February 2003

**Name of Activity:** **Large Card Layout**

**Area:** General: Mathematics Specific: Decimal System

**Materials:** A box containing a full set of large number cards.

**Aims:** Direct: To further develop the Mathematical Mind of the child.

Indirect: To prepare the child to move towards abstraction, to recognize the symbols of the decimal hierarchies.

**Preparation:** Introduction tray for the decimal system, introduction to the decimal cards, association of the beads and cards and previous numeration materials.

**Age:** Second year child **Presentation of the Lesson:**

1. Bring the box of decimal cards to the work mat, which is laid out vertically.
2. Starting with the unit cards, place the 1 card in the top right corner and say “one” as you place it in position.
3. Continue placing and naming the cards 2-9 along right edge of the rug.
4. Lay out the ten cards to the left of the unit column, repeating the language as you lay them down, “one ten, two tens, three tens, etc.”
5. Repeat for hundred and thousands cards.
6. Begin a 3 period lesson. Ask the child to begin by working within one category and always identifying the number at the top of the column first (1, 10, 100, 1000). “Show me 100, 400, 700, etc.”

The children need a lot of practice with this activity to reinforce the right to left sequencing.

**Game #1:**

1. Leave the first cards at the top of each column (1000, 100, 10, 1) but mix up the cards within each category.
2. Ask child to order the cards in proper sequence within each category.

**Game #2:**

1. Mix all the cards within each category and have the child order them properly.

**Game #3:**

1. Leave the first cards at the top of each column (1000, 100, 10, 1) but turn over the cards within each category.
2. Ask child to order the cards in proper sequence within each category.

**Game #4:**

1. Turn over all the cards within each category and have the child order them properly.

**Work of the Teacher:**

Points of Emphasis: Placement of the cards on the mat within columns,

sequencing from right to left. top to bottom, keeping the games fun!

Language: Language for the numbers 1-9,000 **Points of Interest:** Working with large numbers **Work of the Child:**

Points of Consciousness: Zero is a placeholder Control of Error: The teacher

**Variations:** See the games

**Extensions:** Preparation for further work with decimal system operations.

Source: NMI; February, 2003

**Name of Activity: 45 Layout**

**Area:** General: Mathematics Specific: Decimal System

**Materials:** One full set of large decimal cards, one complete set of golden beads and wooden materials for each category, trays with container for unit beads.

**Aims:** Direct: To further develop the mathematical mind.

Indirect: To give the child a concrete, sensorial impression of the decimal system. To prepare the child for later work with the decimal system.

**Preparation:** All previous decimal system work. **Age:** Second year child

**Presentation of the Lesson:**

**Presentation #1: Static 45 Layout**

1. Layout the full set of decimal cards on a mat, leaving spaces between the categories. The full set of thousands is laid out in this work.
2. Point to the one unit card and ask, ”What does this say?” One child fetches one unit bead from the supply and places to the left of the unit card.
3. Continue through the unit category, fetching units to correspond with the number card.
4. Continue fetching categories to match the number cards:

the tens are placed side by side vertically the hundreds are stacked

the thousands are stacked (carefully!)

1. When the 45 layout has been achieved, let the children pause to admire their work and achievement.

**Presentation #2: Dynamic 45 Layout**

1. Layout out the large card layout as in the previous presentation.
2. Point to the unit card and ask “What is this?” “One unit.”
3. Child fetches a unit bead and places it to the left of the number card. Referring to materials, say to child, “This is 1 unit, this says 1 unit.”
4. Say to the child, “If I had one more, I would have 2.”
5. The child fetches another unit bead. Slide the 1 unit down to the 2 card and add the second unit bead and count. Repeat language as above.
6. Repeat the process of sliding down the quantity and adding one more unit until l 9 unit beads have been counted. (Revise and repeat language each time.) Say to the child, ”If I had 1 more unit, I would have 10.”
7. Have child fetch another unit. “10 units are the same as one ten.” Child returns 10 units to the bank and brings back one ten.
8. Count the 10 and place beside (to the left of) the 10 card. “This is ten, this says ten.”
9. Continue as above, sliding tens down and adding an additional ten. Repeat language each time: “This is , this says .”
10. Continue ‘fetching - sliding - counting’ process, making exchange at bottom of categories until 9000 has been put into place.

**Work of the Teacher:**

Points of Emphasis: Reinforcing the language for the decimal system.

Clarity of language with the dynamic layout.

Language: Inherent in the materials. (This is.....; this says....)

**Points of Interest:** Working with the large card layout, fetching the bead material, visual effect of the 45 layout, excitement and satisfaction of doing “big work”.

**Work of the Child:**

Points of Consciousness: Correspondence of bead material to symbol, the bead

material increases in size and weight as the category changes and as the numerical value changes.

Control of Error: The teacher **Variations:**

**Extensions:** Writing out the numbers on a number roll, using colored markers to match the color of the decimal system number cards.

Source: NMI; February 2003

**Name of Activity: Formation of Complex Numbers**

**Area:** General: Mathematics Specific: Decimal System

**Materials:** 3 work mats, trays with container for unit beads, 1 set of large number cards for 1-9, 10-90, 100-900, and 1000.

|  |  |  |
| --- | --- | --- |
| **Aims:** | Direct: | To assist the development of the Mathematical Mind. |
|  | Indirect: | To associate beads with cards, assisting the child in |
|  |  | moving towards abstraction. |

**Preparation:** All previous decimal system work. **Age:** Second year child

**Presentation of the Lesson:**

**Presentation #1: Introduction**

1. Gather together a small group of children.
2. Unroll mat #1 on one side of the room and invite 1-2 children to lay

out the large card layout (only the thousand card is used in this exercise).

1. Place mat #2 in another area of the classroom, sufficiently far to create movement within the exercise. The teacher lays out the bead material equivalent to the number cards on mat #1.
2. Mat #3 is used for combining the materials.
3. Begin the activity by choosing a unit card from mat #1 (6 units).
4. Place the card on the tray. Show the card to the child and say, “This card says 6 units, please go to the mat and fetch 6 units.”
5. Bring child to mat #2 with tray and card to fetch the units. Bring the tray with the units and card to mat #3.
6. Lay the number card on the mat and carefully count the six units, aligning them below the card.
7. Return the units to mat #2 and the card back into the card layout on mat #1.
8. Invite the children to take turns finding a number and fetching the quantities.

**Sequence of the Exercises:**

1. cards to beads working in one category.
2. beads to cards working within one category.
3. cards to beads working with two adjacent categories (units and tens).
4. beads to cards working with two adjacent categories (units and tens).

**Presentation #2: Fetching beads directly from the bank**

1. Prior to the activity, the children are shown the bank and where it is stored on the shelf. The teacher explains that we will now be using wooden 100’s and 1000’s.
2. The activity is similar to Presentation #1 and follows the sequence below:
	1. cards to bead working in two nonadjacent categories.
	2. beads to cards working in two nonadjacent categories.
	3. cards to beads working in three categories.
	4. beads to cards working in three categories.
	5. cards to beads working in all categories.
	6. beads to cards working in all categories.

Reversing process each time helps to reinforce hierarchies and values of the decimal system.

**Work of the Teacher:**

Points of Emphasis: Layout of materials. Reversing the process. Language: Inherent in the materials

**Points of Interest:** Combining bead material with cards, games like quality of fetching the card and “finding” the matching material.

**Work of the Child:**

Points of Consciousness: Numerical values can be represented concretely or

symbolically, they still have the same value.

Control of Error: The teacher

**Variations:** Reversing the process of beads to cards.

**Extensions:** This exercise and the following are laying the foundation for work done at the elementary level. It is important for the child to have full exposure to the decimal materials.

Source: NMI; February 2003

**Name of Activity: Tray of Equivalency**

**Area:** General: Mathematics Specific: Decimal System

**Materials:** A tray containing ten units, ten tens, ten hundreds and one thousand.

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| **Aims:** | Direct: | To further develop the Mathematical Mind. |
|  | Indirect: | To help the child visually perceive the transition to the next |
|  | higher category within the decimal system. To concretely see the |
| relationship between ten of one category being the same as one of |
| the next higher category. |

**Preparation:** All previous decimal system activities. **Age:** Second year child

**Presentation of the Lesson:**

**Presentation #1: Introduction**

1. Count the ten units, placing them in a row to resemble a ten bar.
2. Ask the child if they see anything on the tray that looks the same as the ten units.
3. Place a ten bar beside the unit beads. Move the ten slightly to the left. Place and count all the tens placing side by side to resemble a hundred, counting, “One ten, two tens, three tens.......ten tens.”
4. Ask the child if they see anything on the tray that looks the same as the ten tens. Compare the hundred and then move slightly to the left.
5. Repeat for the hundreds. Stack the hundreds as they are counted. Draw the relationship of the 10 hundreds to the thousand by placing the thousand beside the hundreds and then move slightly to the left.

**Presentation #2: Prepared Tray of Beads**

1. Prepare a tray with a quantity of beads for the children to practice counting and exchanging. For example, the tray would have 15 units, 17 tens and 12 hundreds. This would ensure repeated opportunity to make exchanges.

**Work of the Teacher:**

Points of Emphasis: Ensuring that the child has a clear understanding of

‘equivalence’.

Language: Exchange/exchanging

**Points of Interest:** Counting and making exchanges **Work of the Child:**

Points of Consciousness: 10 unit beads are the same as 1 ten, 10 tens are the

same as 1 hundred and 10 hundreds are the same as 1 thousand.

Control of Error: The teacher

**Variations: Extensions:**

Source: NMI; February 2003

**Name of Activity: Tray of Nines (9’s) or Exchange Tray**

**Area:** General: Mathematics Specific: Decimal System

**Materials:** Tray with 9 unit beads, a tiny box containing 1 unit, 9 tens and 9 hundreds.

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| **Aims:** | Direct: | To further develop the Mathematical Mind. |
|  | Indirect: | To show the child that adding one more in the sequence |
|  |  | implies a change in hierarchy (or category). |

**Preparation:** All previous decimal system work. **Age:** Second year child.

**Presentation of the Lesson:**

1. Gather a small group of children at a mat.
2. Count the units, emphasizing units as you count, “1 unit, 2 units....” up to 9 units.
3. Say to the children, “If I had 1 more unit, I would have 10.” Open the box and reveal the one unit bead.
4. Add to the 9 units to make 10. “What do we know about 10 units?”
5. “10 units is the same as one 10.” Ask a child to bring the units to the bank and exchange for 1 ten.
6. Place the 1 ten on the rug and count the additional 10’s from the tray. Draw the children’s attention to 10 tens being the same as 1 hundred.
7. Ask a child to take the tens to the bank and exchange for 1 hundred.
8. Repeat above counting and exchanging of 10 hundreds for 1 thousand.
9. When the activity is complete there will be only one thousand on the mat.

**Work of the Teacher:**

Points of Emphasis: Reinforcing the language for the category being counted. Language: Inherent in the materials.

Points of Interest: Adding the unit stored in the tiny box. Process of exchanging to the next higher category.

**Work of the Child:**

Points of Consciousness: If you add one more to a sequence consisting of 9

you automatically transfer to the next higher category in the decimal system.

Control of Error: The teacher

**Variations:** **Extensions:**

Source: NMI; February 2003

**Name of Activity: Mystery Quantity Exchange Tray**

**Area:** General: Mathematics Specific: Decimal System

**Materials:** A tray with a random selection of golden beads.

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| **Aims:** | Direct: | To further develop the child’s Mathematical Mind. |
|  | Indirect: | The child will be able to turn an unknown quantity of beads |
|  |  | (through exchanging) into a reduced, simple number. |

**Preparation:** Bank game, Exchange tray, Tray of nines. **Age:** Second year child.

**Presentation of the Lesson:**

1. The teacher prepares a tray with a random mix of units, tens, and hundreds.
2. The child counts the beads and makes exchanges as needed.
3. The child can either write the number into a booklet or express the quantity with the large cards.

**Work of the Teacher:**

Points of Emphasis: Teacher keeps changing the amount of beads. Language: Category, exchanging.

**Points of Interest:** Reducing the quantities within a category by exchanging.

**Work of the Child:**

Points of Consciousness: By exchanging, the quantities within categories can be reduced.

Control of Error: The teacher.

**Variations:** Change the quantity of beads. Have result be 1,000.

**Extensions:** Record quantity into a mystery Quantity Booklet. Source: NMI; February 2003

**Name of Activity: Exchange Game**

**Area:** General: Mathematics Specific: Decimal System

**Materials:** A tray with 1 thousand, 20 hundreds, 20 tens and 20 units. Three large dice (1 green, 1 blue and 1 red), Two or three mats and several charts which are narrow strips divided into the 4 categories; units, tens, hundreds, thousands.

|  |  |  |
| --- | --- | --- |
| **Aims:** | Direct: | Child will be building their Mathematical Mind and |
|  |  | learning to take turns. |
|  | Indirect: | Child will learn how to read numbers on a dice and retrieve |
|  |  | that quantity. They will be reinforcing their skills of |
|  |  | exchanging into the next category. |

**Preparation:** Previous experience with exchanging. **Age:** First to Second year

**Presentation of the Lesson:**

1. Have each child arrange their mat with the control strip at the top. The control chart helps the child remember what place they’re in.
2. Choose a child to go first. Child rolls the green die and takes that amount of units and puts it under the unit place.
3. The other children take turns with the green die. Keep using the green die until someone needs to make an exchange (10 units for 1 ten).
4. Once an exchange to tens has been made, the child uses the blue die.
5. Children take turns getting tens until 100 is exchanged for, then begin using the red die.

5. Continue in this way until someone reaches one thousand. The game is over. The children can put the game away or play again.

**Work of the Teacher:**

Points of Emphasis: The teacher models taking turns and counting beads. Language: die, dice, exchange

**Points of Interest:** Rolling the dice and playing a game. **Work of the Child:**

Points of Consciousness: Child will reinforce their understanding of

exchange. Child will experience winning a game and loosing at times.

Control of Error: The control chart helps the child keep his place.

**Variations:** Vary the size of the dice.

**Extensions:**

Source: NMI; February 2003

Decimal System ~ Operations

Static and Dynamic Addition

Static and Dynamic Multiplication

Static and Dynamic Subtraction

Static and Dynamic Division

**Name of Activity: Operations: Static and Dynamic Addition**

|  |  |  |  |
| --- | --- | --- | --- |
| **Area:** | General: | Mathematics |  |
|  | Specific: | Decimal System: | Operations |

**Materials:** A supply of beads in the “bank” (50 each of units, tens, hundreds and thousands), one set of large number cards (1-9000), three sets of small number cards (1-3000)\*, three trays with unit cups. \*(Small cards through 9000 can be used in the dynamic example.)

|  |  |  |
| --- | --- | --- |
| **Aims:** | Direct: | To further develop the Mathematical Mind and to appreciate |
|  |  | the complexity and simplicity of our numerical system. |
|  | Indirect: | To introduce the child to the process of addition using the |
|  |  | golden bead materials. To show the child how numbers can |
|  |  | carry from one hierarchy to the next. |

**Preparation:** All previous decimal system work. **Age:** Second year child.

**Presentation of the Lesson:**

**Presentation #1:** Static Addition

1. Gather a small group of children at a rug. Give each child a small set of cards and ask them to lay out their cards on a small rug.
2. The teacher lays out the large number cards and a large mat for the operation.
3. Gather the children together and assign a number to each child, being sure that there will be no exchanges within the categories (static problem).
4. The teacher asks the children to go to the bank and “fetch” their numbers. Each child gets their number and brings it to the teacher to check. (e.g., 1433 + 2312 + 3124)
5. As the quantities are checked, the teacher helps the children to orient their beads, placing the categories right to left on the operations rug.
6. Begin by asking the first child, “What did you bring?” Guide the child by asking, “How many units did you bring?”
7. Child counts the unit beads and the teacher helps the child to orient the unit beads near the top of the operations rug.
8. Ask the first child, ”What number says units?” Place the unit number card below the unit beads.
9. Help the child to count out the tens, placing these to the left of the unit beads. Place the ‘tens’ number card below the ten bars.
10. Continue to identify the hundreds, and then the thousands, moving from right to left and placing the number cards below the beads.
11. When all the beads have been counted, the teacher states, “You brought 1433.” After identifying the child’s number, place the cards together and move them to the right of the beads, along the edge of the mat.
12. Repeat the above process with each of the other children. Each new child’s beads are placed vertically below the previous child’s beads. (The small cards are first placed beneath the beads, and then combined and moved to the right of the rug.)
13. When all the number cards and corresponding quantities have been laid out, begin the process of addition.
14. Ask the first child to slide down the unit beads to the bottom of the work mat and to count the units.
15. Next, invite the child to get the large number card corresponding to the quantity of units and to place the card beneath the beads.
16. Ask the second child to slide down the tens and get the corresponding large number card. Place the card beneath the quantity of tens.
17. Repeat for the hundreds and then the thousands categories.
18. Gather up the large number cards for the “magic slide.” Say to the children, “If we take (first child’s number: 1433) and (second child’s number: 2312) and (third child’s number: 3124) and put them all together, we get (answer: 6869).” Hold the cards at top and bottom and slowly tap cards as you say the answer, so that the children see the numbers appearing as you say them.
19. End the lesson by saying, “When we put (add) numbers together, they are called addends. This is called addition. When the addends are put together, we call that the sum. The sum is also called the answer.”

Note: A red plastic plus sign and a long plastic line can be used to indicate addition and to separate answer.

**Presentation 2:** Dynamic Addition (example involves small cards to 9000;

e.g., 3474 + 1653 + 4384 = 9511)

1. Dynamic Addition is presented after the child has had a thorough introduction to static addition and when they understand the process of exchange.
2. The children lay out small number cards while the teacher lays out the large number cards and a work mat.
3. The teacher gives the children numbers which will require exchanges in several categories when added. (Except in the thousands!)
4. The children “fetch” their quantities from the bank and arrange their beads and number cards as in the above lesson, placing the categories right to left with units on the right.

(As the quantities are counted, the small number cards representing each category of beads is first placed below the row of beads. After all of a child’s beads are counted, the small cards are combined and

moved to the right of the child’s beads, near the right edge of the mat.)

1. When all numbers are laid out, begin the process of addition by asking a child to add the units by pulling them to the bottom of the mat. Ask the child to count the units.

If there are more than 9 units, pause to determine if the child understands the need for an exchange. If needed, guide the child through the process, “What could we do with 13 units? Yes, we could exchange 10 units for 1 ten!” (This may come as an exciting discovery for some children. Remember to share in their excitement!)

1. Continue counting the remaining categories. Make exchanges as needed.
2. Gather up the large number cards for the “magic slide.” Say to the children, “If we take (first child’s number: 3474) and (second child’s sumber: 1653) and (third child’s number: 4384) and put them all together, we get the sum of 9511.

Hold the cards at top and bottom and slowly tap cards as you say the answer so that the numbers appear as you say them.

1. End the lesson by reinforcing the language of addends and sum.

Note: Red plastic plus sign and a long plastic line can be used as before.

**Work of the Teacher:**

Points of Emphasis: Methodical layout of small and large cards.

Placement of cards and materials on mat so that the children can see the process of addition concretely. Verbalizing the process at the end of the lesson.

Language: Addition, exchange, units, tens, hundreds, thousand and other language relevant to the lesson.

**Points of Interest:** Working as a group, working with the bank, seeing numbers grow when added together, the ‘magic slide’.

**Work of the Child:**

Points of Consciousness: Addition is a process where numbers are combined,

resulting in a new, larger number. Ten of one category is equivalent to one of the next higher category.

Control of Error: The teacher.

**Variations:**

**Extensions:** Recording the problems into a notebook.

Source: NMI; February 2003

**Name of Activity: Operations: Static and Dynamic Multiplication**

|  |  |  |  |
| --- | --- | --- | --- |
| **Area:** | General: | Mathematics |  |
|  | Specific: | Decimal System: | Operations |

**Materials:** A supply of beads in the “bank” (50 each of units, tens, hundreds and thousands), one set of large number cards (1-9000) three sets of small number cards (1-3000), three trays with unit cups.

|  |  |  |
| --- | --- | --- |
| **Aims:** | Direct: | To further develop the Mathematical Mind and to appreciate |
|  |  | the complexity and simplicity of our numerical system. |
|  | Indirect: | To introduce the child to the process of multiplication |
|  |  | using the golden bead materials. To show the child how |
|  |  | numbers can carry from one hierarchy to the next. |

**Preparation:** All previous decimal system work. The child should be secure in the process of addition using the decimal system materials.

**Age:** Second year child. **Presentation of the Lesson:**

**Presentation: Static Multiplication** (e.g., 2312 x 3 = 6936)

1. Gather a small group of children (3) and tell them we are doing something new today. Ask them to each lay out a small set of number cards on a table or work mat.
2. The teacher lays out a large set of cards and a work mat for the beads.
3. The teacher goes with each child to select a number for the operation and she gives each child, the same number (2312).
4. Each child fetches the appropriate quantities and lays them on the rug as they have done previously for addition. (There may be some form of initial reaction on the part of the children, as they discover that they each have the same quantity. Do not explain multiplication at this point, merely say, “Yes, that’s different, isn’t it.”)
5. The children bring down the bead material and count category by category, fetching the corresponding large number card.
6. When all the categories have been added together say, “When you took (first child’s number) and (second child’s number) and (third child’s number) and put them together you made (final number, or product).
7. Summarize the work: “If you take 2312 three times you make 6936. When we add the same number, we call this multiplication.”

Notes: At a later date, you can show the children how to write this equation:

(using small cards). Start with the three identical sets of small cards representing the multiplicand, placing one above the other. The teacher emphasizes that the number ‘taken’ only needs to be written once, so two sets of small cards representing the number can be removed. “We do need to show ‘how many times we’ve taken’ this number.” Get a small 3 and introduce the ‘times’ sign, placing these beneath the remaining multiplicand cards, aligning the 3 beneath the units. The large answer cards (product) are placed beneath this.

A yellow multiplication sign and a yellow ‘equals’ strip can be used to separate the equation from the sum.

Introduce the terms multiplicand, multiplier and product as the child seems ready.

**Presentation #2: Dynamic Multiplication** (e.g., 1543 x 3 = 4629)

1. This process is the same as Dynamic Addition. Here the children are assigned the same number. As the children count the categories, pause when they reach 10 to see if they remember to exchange. If not, guide them by asking, “What do we know about 10 units?”
2. The teacher summarizes the process at the end of the lesson.

**Presentation #3: Multiplication using Skittles** (e.g., 1324 x 4 = 5296)

1. A child may want to work on their own with a problem with a multiplier higher than 3. Show the child how to use skittles to represent the number of times they are taking the number. The skittles are laid out top to bottom along the left side of the rug. The child places the beads to the right of the skittles each time the number is brought back from the bank. Combine and exchange each category as before.

**Work of the Teacher:**

Points of Emphasis: Methodical layout of the materials, to keep the

categories straight and the process of multiplication clear. Summarizing the process at the end of the lesson.

Language: Multiplicand, multiplier, product, exchange and other relevant language.

**Points of Interest:** Working together in a small group, seeing the

number grow, using skittles. The number (multiplicand) need only be written once in multiplication.

**Work of the Child:**

Points of Consciousness: Multiplication is a process of addition. Ten of one

category is the same as one of the next higher category.

Control of Error: The teacher.

**Variations:**

**Extensions:** Recording work.

Note: A yellow plastic multiplication sign and a long plastic line can be used to indicate multiplication and to separate the answer (product).

**Name of Activity: Operations: Static and Dynamic Subtraction**

**Area:** General: Mathematics Specific: Decimal System

**Materials:** A supply of beads in the “bank” (50 each of units, tens, hundreds and thousands), one set of large number cards (1-9000) three sets of small number cards (1-3000) two for the children’s minuend and one set to use for the final answer, three trays with unit cups.

|  |  |  |
| --- | --- | --- |
| **Aims:** | Direct: | To further develop the Mathematical Mind, to foster an |
|  |  | appreciation for the complexity and simplicity of |
|  |  | mathematics, to move the child towards abstraction. |
|  | Indirect: | To gain an understanding of the concept of subtraction |
|  |  | and to gain an impression of the operation of subtraction. |

**Preparation:** All previous decimal system work. **Age:** 4-5 years old

**Note on Presentation:** After initial lesson, only subtract one subtrahend from minuend. **Presentation of the Lesson:**

**Presentation #1: Static Subtraction**

1. Invite two children to the lesson and ask each child to lay out a set of small number cards.
2. The teacher lays out the large number cards and one set of small number cards, along with a work mat for the operation.
3. Explain to the children that we are doing something new today. “I am thinking of a number.” Lay out the large cards for 9743. Fetch the quantities for this number.
4. Ask the children to help count and check the quantity, starting with units.
5. The teacher assigns numbers to the children and asks them to get the corresponding numbers from the small card layout. The first child is given 1211 and the second child is given 3221.
6. Say to the children, “I have such a large number that you can take your numbers away from my materials today.”
7. Begin the process of subtraction with one child’s number, starting with the units. As the child takes away their beads from the larger number, these are placed in categories at the bottom of the rug with the cards beneath the beads.
8. Say to children, “My number has changed. Now I have .” Turn over large (minuend) cards and place new large cards beneath turned over cards.
9. Repeat ‘taking away beads’ with the second child (beginning with units).
10. Say to the children, “I had a large number to start with. Do I still have the same number? No, now I have much less.” Count the teacher’s remaining beads starting with the units and fetch the small number cards that correspond to teacher’s remaining beads. (“We are using the small cards because my number has gotten smaller.”)
11. Summarize the process, beginning by turning original minuend face-up and placing ‘adjusted’ minuend face-down. “We started with 9742 and we took away 1211, then we took away 3221, and we were left with a new number: 5311. When we take numbers away we call that subtraction.”

Note: Original number is minuend, number taken away is called the subtrahend and the answer is the difference.

**Presentation #2: Dynamic Subtraction**

1. The layout and process is the same as for static subtraction except there will be exchanges.
2. Make a number with the large cards: 9737. Fetch the corresponding quantities from the bank and bring to the rug.
3. Check that the correct quantities were brought.
4. Assign a number to each child (3150) and (2836). Each child gets small number cards for their numbers.
5. Begin by taking away one child’s number, starting with the units. The highlight is when you don’t have enough beads. Involve the children in determining how to proceed and in making the exchange.
6. After completing first child’s number, turn over minuend and place new large cards beneath original minuend as before. Continue by taking away second child’s number, exchanging as necessary.
7. Identify remaining amount with small cards. When summarizing the process, change back the original minuend cards as before. “We started with 9737 and we took away 3150 and 2836 and we ended up with 3751.”

**Work of the Teacher:**

Points of Emphasis: Methodical layout of the materials, emphasizing the taking away process.

Language: Minuend, subtrahend, difference, borrowing, exchanging and other language appropriate to the lesson.

**Points of Interest:** Working with the decimal system, learning the process of subtraction.

**Work of the Child:**

Points of Consciousness: Subtraction is a process whereby a number is reduced in size.

Control of Error: The teacher.

**Variations:** The original number is all given out.

**Extensions:** Recording the problems onto paper.

Note: A green plastic plus sign and a long plastic line can be used to indicate subtraction and to separate the answer (the difference).

**Name of Activity: Operations: Static and Dynamic Division**

**Area:** General: Mathematics Specific: Decimal System

**Materials:** A supply of beads in the “bank” (50 each of units, tens, hundreds and thousands), one set of large number cards (1-9000) three sets of small number cards (1-3000), three trays with unit cups. Nine green skittles.

|  |  |  |
| --- | --- | --- |
| **Aims:** | Direct: | To further development the Mathematical Mind and to |
|  |  | increase awareness of the beauty of mathematics. |
|  | Indirect: | To gain an understanding of the concept of division, to gain a |
|  |  | concrete impression of the operation of division and to move the |
|  |  | child towards abstraction. |

**Preparation:** All previous decimal system work **Age:** 4-5 year olds

**Presentation of the Lesson:**

**Presentation #1: Static Division** (e.g., 9369 ÷ 3)

1. Gather together a small group of children (3) and invite each child to lay out a small set of cards.
2. The teacher lays out a large set of number cards and a work rug.
3. The teacher selects a number (9369) and lays the cards out on the rug.
4. The teacher and the children go to the bank and fetch the corresponding quantities. Check the quantities and arrange the beads and cards on the rug as done in previous activities.
5. Say to the children, “I’m going to share this number so that each of you get the same amount.” Introduce the skittles as representing each individual child.
6. Start by sharing the 1000’s first.\* You can say to the child, “I’m going to give you biggest amount first. What do you think that would be?”
7. “I had enough to share between you equally.” Have children each get a small number card to identify amount.
8. Repeat the process with the hundreds, tens and units, getting answer card before moving on to next category.
9. Ask one child how many thousands they have, hundreds, tens and units. The child gets a small number card for each category.
10. Repeat the same process with the second and third child.
11. “I shared my number with you, 9369, and you each got 3123. Our answer is what one person got. When we share equally, we call this division.”

**Presentation #2: Dynamic Division** (e.g., 1743 ÷ 3)

1. The layout for the materials is the same as for static division.
2. The teacher selects the number to be divided, making sure it will involve several exchanges within categories, but will come out evenly. Example: 1743 (divided by 3). Set out skittles to represent divisor.
3. Begin by sharing the thousand, discovering that you cannot share one thousand equally. Guide the children in a discussion to determine what we can do? Exchange the thousand for 10 hundreds.
4. Share the ‘exchanged’ hundreds and the original hundreds to the children. Child gets answer card for hundreds.
5. Exchange the remaining hundreds for tens and continue the process with the tens and units, getting answer card each time.
6. Summarize the process and review the number cards.

**Work of the Teacher:**

Points of Emphasis: Starting the process of sharing with thousands, clear

layout of the materials and establishing clear steps.

Language: Inherent in the lesson (dividend, divisor, quotient) **Points of Interest:** Exchange categories for sharing.

**Work of the Child:**

Points of Consciousness: Division is a means whereby a larger number can be

broken down into smaller quantities.

Control of Error: The teacher

**Variations:**

**Extensions:** Record problems onto paper. Introduce a division problem that involves a remainder.

Note: A blue plastic division sign and equals sign can be used, although the written form for division is not the same as the preceding operations.

\* Division is performed beginning with the largest category to avoid returning to prior categories to make repeated exchanges and sharing of exchanged amounts. It is possible to complete the work by doing all categories randomly. (Dr. Montessori describes doing multiplication on the bead frame in a random order in her book, The Advanced Montessori Method.)

Memorization

Snake Game for the Research of 10

Positive Snake Game

Addition Strip Board

Negative Snake Game

Negative Strip Board

Multiplication with Bead Bars Multiplication Bead Board Unit Division Board

Finger Charts~

Addition

 Subtraction

 Multiplication

Division

**Name of Activity: Snake Game for the Research of 10**

**Area:** General: Mathematics Specific: Memorization

**Materials:** Nine gold ten bars, two each of the colored bead bars 1-9, a small black felt mat, a container for the golden tens and a separate container for the colored beads, a tray for all the materials. Optional: a snake skin stored in a basket or box.

**Aims:** Direct: To develop logic, order and reasoning, to further develop

the Mathematical Mind.

Indirect: To research 10 and the combinations to make 10 which is preparing the child for addition.

**Preparation:** All previous numeration work **Age:** Second to Third year child **Presentation of the Lesson:**

**Presentation #1:**

1. Carry the tray to the table and place to the top of the work space.
2. If you have s snake skin, you can enchant the child by telling a story revolving around how you found the skin, emphasizing that the snake changed its skin and that you are going to show the child how she can make a snake using the colored bead bars and change its appearance.
3. Unroll the felt mat and lay in front of the child. Invite the child to take out the colored bead bars.
4. Invite the child to build a bead stair. As the child is building their bead stair, the teacher builds a second bead stair in reverse order (if the child builds a stair with 1 on the top of the pyramid, the teacher builds a stair with 1 at the bottom.
5. Tell the child that we are going to build a colored snake.
6. Start to build the snake in even pairs of 10’s (take the 1 bar from the child’s bead stair and the 9 from the teacher’s bead stair.) Continue building in pairs: 2 & 8, 3 & 7, etc.
7. Stop to admire the beautiful snake.
8. Bring down the box of 10 bars and tell the child that we are going to change the colored snake into a golden snake.
9. Begin to count the snake, counting each bead up to 10. Pause and place a ten beside the bead combination.
10. Take the 1 & 9 bars away and place into a discard pile to the left.
11. Move the 10 bar into position aligned with the snake. Continue counting the combinations of 10, pausing and replacing with a 10 bar.
12. Pause to admire the beauty of the golden snake. Invite the child to build the snake again if interested.

**Presentation #2: Checking**

1. Invite the child to build a colored snake as before (controlled combinations of 10) and change it into a golden snake.
2. Tell the child that we are going to check our work by lining up the 10 bars and comparing them to the colored bead bars.
3. Line up the 10 bars across the mat. Place the bars vertically with space

in between. Arrange colored bars at bottom of mat from longest to shortest.

1. Place a 9 bar beside the first 10 bar and count to 9. Ask the child how many more beads do we add to 9 to make 10. Add a 1 bar below the 9 bar.
2. Continue in this manner until all the colored beads have been matched to the golden beads to make combinations of 10.

**Work of the Teacher:**

Points of Emphasis: Building the bead stair in reverse order so that the

combinations amount to 10. Being dramatic in the telling of the story of the snake skin. Careful placement and counting of the beads.

Language: Numbers 1-10, exchange, snake, check.

**Points of Interest:** Making a colored snake and changing it into a golden snake.

**Work of the Child:**

Points of Consciousness: Combinations of numbers that make 10. Control of Error: Sufficient beads to complete the exercise.

**Variations:** **Extensions:**

**Name of Activity: Positive Snake Game**

**Area:** General: Mathematics Specific: Memorization

**Materials:** A box containing 5 colored bead stairs, a box with 25 golden 10 bars, a set of black and white bead bars 1-9 in a box, a red felt mat, a small notched counter, a tray for the materials.

**Aims:** Direct: Deepen an appreciation for our number system and to

further develop the Mathematical Mind.

Indirect: To give the child practice in memorizing the essential addition combinations.

**Preparation:** Numeration work, teens and tens, decimal system including exchanging. **Age:** Second to Third Year Child

**Presentation of the Lesson:** **Presentation #1:**

1. Explain to the child that we are going to do some new work with the snake game.

2, Introduce the black and white beads and explain that they are called “keepers” because they help to keep our place when counting the snake.

1. Build the black and white bead stair.
2. Bring down the box of colored beads and begin to build a colored snake. Keep bead bar box closed after building snake. (6+8+7+5+2+9+1+4+5+3)
3. Tell the child that we are going to change our snake into a golden snake like we have done before.
4. Begin to count the snake, if it is not an exact combination of 10, insert the counter after the 10th bead. (Initially, it is in the middle of the 8.)
5. Place a 10 bar above the bead combination and count the remaining beads on the bar after the 10th bead. Find the corresponding black

and white bead bar (the black 4) and align into the snake, removing the colored beads and placing them in a discard pile to the bottom left.

1. Count to 10 again starting with the black and white beads. Replace the colored bead bars with a ten bar and, when necessary, use the black and white “keepers”. When discarding the black and white bead bars, place them back into the “keeper” stair for further use.
2. Repeat until colored snake has been replaced by a golden snake. If the final segment of the snake is less than 10, use the black and white remainder beads to represent this amount.

**Presentation # 2: Checking**

1. Lay out the golden 10 bars across the mat, leaving space in between each one. Line up the colored bead bars ranked from highest number to lowest in the lower left of mat.
2. Lay a 9 bar beside the 10 bar. Ask child what to add to 9 to make 10. In this example, all combinations can be found to make ten, however:

The Child may need to exchange bead bars to find the proper quantity. For example if you need a 3, bring down a 4 bar and place in front to the bead bar box. Find the combination needed (a 3 bar and a 1 bar). Place the 4 bar back into the box and replace cover.

If a black and white bead was used as a remainder in the snake, then it will be used in the check and compared to colored beads as with the golden bead bars.

**Presentation #3: Isolating Combinations**

1. Invite the child to build the black and white bead stair. Now invite the child to build a long colored snake. (6+8+7+5+2+9+1+4+5+3)
2. Bring down the first 2 bead bars and place them in a vertical position in front of the child. Count to 10, bringing down a 10 bar and if needed, a black and white bar for any beads past ten. (In this case the black 4.)
3. Place the 10 bar back into the snake, along with any keeper bars (ten on left of keeper). Place discarded colored bars below for check.
4. Continue to isolate bead combinations (two colored bars or keeper bar and colored bar) and changing the snake into a golden snake.
5. Check the work as in previous presentations.

**Presentation # 4: Long Snake with Multiples**

1. Build a snake with combinations of the same number. (5+5+5+3+3+7+7+7)
2. Check work as in previous presentations.

**Work of the Teacher:**

Points of Emphasis: Clarity of the presentation, layout of the beads and

bead stair, checking the work.

Language: The language is within the lesson.

**Points of Interest:** Excitement of changing the colored snake into a golden snake. Making exchanges with colored bars to make check come out even.

**Work of the Child:**

Points of Consciousness: Combinations of 10, the golden snake has the same

number of beads as the colored snake.

Control of Error: The work is checked after snake has been changed.

**Variations:** Child builds a long snake with multiples of the same number, a 5 snake or a 4 snake, etc.

**Extensions:**

**Name of Activity: Addition Strip Board**

**Area:** General: Mathematics Specific: Memorization

**Materials:** A board divided into eighteen squares across from left to right and twelve squares from top to bottom. Each square is 2 cm x 2 cm. The grid is blue and a vertical red line divides the board. Above the grid are the numerals 1-18. The numerals 1- 10 are in red. The numerals 11-18 are in blue.

Two sets of wooden numbered strips, one set is blue with a numeral (1 - 9) in red at the end of the strip. The other set is red, divided into squares by blue lines, with a numeral in blue (1-9) at the right end of each. Prepared addition forms, a pencil and additional control charts 1 & 2 for checking work.

**Aims:** Direct: To develop the Mathematical Mind of the child through logic, reason and ordering.

Indirect: To give the child all the possible combinations in addition and to assist the child in memorizing the combinations.

**Preparation:** Numeration activities, teens and tens, decimal system work and the snake games. **Age:** Approximately third year child.

**Presentation of the Lesson:**

**Presentation #1: Initial Presentation**

1. Invite the child to the lesson and carry the work to a table.
2. Remove the blue strips and place on the table randomly. Build a stair above the board on the left (one at bottom with left edge aligned).
3. Repeat with the red strips, building them to the right of the blue strips (from 1 - 9 upwards).
4. Show the child how to use the strips by placing the blue 1 strip on the board. aligning it in the designated square. Place the 1 red strip beside it and say that 1 + 1 = 2.
5. Replace the red 1 strip to the stair and take the red 2 strip and place on the board beside the 1 blue strip: 1 + 2 = 3.
6. If the child is interested, show them the math booklets which have been prepared. Show the child how to read the equation and place the blue and red strip on the board to find the answer.
7. Encourage the child to complete the whole booklet over time.

Note: The blue strips represent the table that the child is working with, giving the impression of a whole number. The red strips represent the number being added, the lines showing ‘quantity’.

**Presentation #2: Combinations of One Number**

1. Lay out the materials as described in the previous presentation. Child will also need a tray with paper, ruler and a pencil.
2. Ask the child to think of a number between 1 and 9. For example: 7. Write 7 at the top of the paper.
3. Say to the child, “How many ways can we get to 7?” Start with the 1 blue strip, placing it on the board. “How many more do we need to make 7?” Count the squares if necessary. Find and place the red 6 strip on the board.
4. Record onto paper: 1+ 6 = 7.
5. Leave this combination on the board. Place the 2 blue strip underneath. Count and place the 5 red strip on the board. Again, leave the combination on the board.
6. Continue in this manner until all the addition facts for 7 have been explored and recorded. (7 + 0 is not included.)

**Presentation #3: All combinations of a Number in Order**

1. The child is familiar with making combinations. For this exercise the child is using graph paper.
2. Lay out the materials as described previously.
3. Tell the child that we are going to make a lot of combinations. We are

going to make an addition chart. Start with 2 (there are no combinations for 1).

1. Place the 1 blue strip on the board. “How many more to make 2?” Child places the 1 red strip on the board to make the combination of 1+1=2. Ask the child if there is another way to make 2.
2. Record the combination on the paper and replace the strips.
3. Explore the combinations for 3 (1 + 2 = 3 and 2 + 1 = 3). Record combinations and replace strips.
4. Continue exploring all the positive combinations for numbers up to 9.

Note: Zero is not demonstrated in this exercise. The focus is on the possible combinations to make a number.

**Presentation #4: Commutative Law (Order Principle)**

1. Paper and pencil are required for this lesson.
2. Lay out the material for the lesson. Think of a number (7). Write the number at the top of the page.
3. Place the 1 blue strip on the board. Now place the 6 red strip on the board to make 7.
4. Now place the 6 blue strip on the board and the 1 red strip and say, “We have made a discovery; 1+6=7 and 6+1=7.” Record the equations.
5. Explain that this is called the commutative law. (It is sometimes called the Order Principle - the order of the numbers added does not matter.)

**Work of the Teacher:**

Points of Emphasis: Careful layout of the materials, clear, simple language so

that the child understands the concept of addition.

Language: Woven into the lesson.

**Points of Interest:** Discovering new work, discovering combinations to make numbers.

**Work of the Child:**

Points of Consciousness: Numbers can be added together, the same numbers

added in reverse order give the same answer.

Control of Error: The answers can be found at the top of the board.

**Variations:** **Extensions:**

Note: There are other versions for laying out the strips. Consistency within the school or classroom may take precedent. The above placement reflects the layout of the number rods (9 on top, 1 on bottom, left edge aligned).

**Name of Activity: Negative Snake Game**

**Area:** General: Mathematics Specific: Memorization

**Materials:** Boxes containing colored bars (about 3 of each), 20 gold ten bars, a black and white bead stair, a negative bead stair (gray), a tray for the materials, a small notched card or bridge for counting and a green mat.

|  |  |  |
| --- | --- | --- |
| **Aims:** | Direct: | To create an awareness of the beauty of mathematics and |
|  |  | its simplicity and complexity. |
|  | Indirect: | To familiarize the child with subtraction combinations. |

**Preparation:** Number rods and decimal system work. **Age:** Third year child.

**Presentation of the Lesson:**

1. Bring the materials to a table and arrange the black and white bead stair to the center right. Build a gray bead stair to the left of this.
2. Construct a snake using the colored beads and insert gray bead bars into the snake. Ensure that a reasonable number of colored bead bars are included in the snake before placing any of the gray bars. Example: 9 + 4 - 9 + 6 - 7 + 8 - 6
3. Start to change the snake into a golden snake (as done in the positive snake game).
	* count to ten, using the bridge or your finger, at the tenth bead exchange for a 10 bar, count the remainder and place a keeper bead (3) into the snake (black and white beads).
	* set aside the colored beads for checking.
	* continue to count the snake, when a gray bar is reached explain to the child that the gray beads tell us to take away.
	* count the beads on the gray bar (begin on the right), then count back that many beads from the keeper and ten (right to left).
	* place bridge at the last bead to be taken away. If there is a remainder behind the bridge, count the number of beads remaining and place a keeper bar (4) beside these beads.
	* take away the gray bar (place to the left below for later checking); the ‘used’ keeper (return to keeper stair); and, in this case, the ten bar (return to box of tens).
	* move new keeper bar (4) up to next remaining bar in snake (colored 6).

Note: return any golden bead bars back into the supply box and keeper bars to the keeper stair. Remove grey bars and colored bead bars and place to the left of the mat for later checking (do not return to supply bead box).

* + if any keeper bars were replaced, place them back into the black and white stair.
1. Continue to change the snake in this manner. When the snake has been transformed into a golden snake, there may be a black and white remainder bar (this should be included in the checking process).
2. The child may notice that the snake is shorter than the original snake.

Checking the Snake:

* + line up the colored bead bars in ordered groups (largest to smallest).
	+ line up the gray bead bars and the golden bead bars.
	+ match combinations starting with the gray bars and then the golden bead bars.
	+ make exchanges of colored bars as before if necessary.

**Work of the Teacher:**

Points of Emphasis: Placing the gray beads into the snake, counting carefully

and counting backwards when a gray bar is reached.

Language: Within the lesson.

**Points of Interest:** Discovering a new snake game, learning about subtraction, seeing the snake get shorter.

**Work of the Child:**

Points of Consciousness: Subtraction means that something is made smaller. Control of Error: The snake is checked.

**Variations: Extensions:**

**Name of Activity: Negative Strip Board**

**Area:** General: Mathematics Specific: Memorization

**Materials:** A board divided into eighteen squares across from left to right and twelve squares from top to bottom. Each square is 2 cm x 2 cm. The grid is red and a vertical blue line divides the board. Above the grid are the numerals 1-18. The numerals 1-9 are blue. The numerals 10-18 are red.

A set of neutral wooden strips representing 1 through 18. Two sets of numbered wooden strips as with the addition strip board. One set is blue with a numeral (1-

9) in red at the end of the strip. The other set is red, divide into squares by blue lines, with a numeral in blue (1-9) at the right end of each. Prepared subtraction booklets, a pencil and control charts 1 & 2.

**Aims:** Direct: To further develop the Mathematical Mind and the

development of the intellect.

Indirect: To familiarize the child with subtraction combinations.

**Preparation:** Numeration work, decimal system operations. **Age:** Third year child.

**Presentation of the Lesson:**

**Presentation #1: Brown and Blue Strips**

1. Introduce the child to the material and bring it to a work space.
2. Have the child build the brown strips to the right above the board, beginning with the shortest strip and aligning the left edge.
3. Explain that for this work we are only using the blue strips and not the red strips. Build the blue strips above the board to the left of the wooden strips, with the one on the bottom and aligned on the left side.
4. Show the child the booklet reviewing the subtraction symbol.
5. The first problem in the booklet reads 18 – 9. Place the blue 9 strip over the numbers on the right. “How many are left?” The answer is always at the end of the blue strip. 18 – 9 = 9.

(\*Note: When using blue strips only, strips are placed over numbers on the board. In following lessons when using red and blue strips, only the brown strip covers numbers on board and colored strips go on grid.)

1. The next example in the booklet reads 17 – 9. Show the child how the brown strips are used to cover the numbers at the top of the board which are not going to be used. Cover the number 18 with the shortest brown strip so that the grid only shows numbers up to 17.
2. Place the blue strip of 9 to the left of the brown strip. Read the answer: 17 – 9 = 8. Record the answer into the booklet.
3. Replace the blue 9 strip with the blue 8 strip for the next equation and record your answer: 17 – 8 = 9.
4. Continue in the same manner, replacing the blue and brown strips as above until the child is comfortable working independently.

**Presentation #2: Building the Tables (Combinations of Numbers)**

1. For this exercise the child uses both the red and blue strips. Blue and brown strips are built as before, with the red strips built above the board to the left (placement of colored strips is opposite of positive strip board for addition).
2. Answers are recorded on a single sheet of paper.
3. Show the child how to build a combination table: Combinations for 8:
	* Use brown strips to cover numbers after 8 and write 8 at the center top of paper.
	* Start to make combinations for 8 by placing red 8 strip in upper left corner of grid. (red strip is not placed over remaining numbers on board.)
	* Place the red 7 strip below the 8 red strip to the left edge of

grid and ask child how many more do we need to make 8? “One.”

* + Place the 1 blue strip at the right edge of the red 7 strip.
	+ Continue in this manner until all the combinations have been placed, ending with placing the blue 8 strip.
	+ “You have built the subtraction table of 8.”
	+ The child records the combinations. They should be recorded in a tabular form, the subtrahend (the number being subtracted) is on the blue strip and the answer (difference) is on the red strip.
	+ Show the child how to begin the subtraction process by taking away the blue 8 strip at the bottom of the strips.
	+ Ask the child how many are left. “None” Say the problem and record on page (“Eight take-away eight is zero.”)
	+ Take away the blue strip of 7 and ask how many is left. “One.” Say the equation and record answer on paper.
	+ Continue removing blue strips, saying equation and recording, until you reach the combination 8 – 0 = 8.
	+ The written table of the above example will appear as:

8

8 – 8 = 0

8 – 7 = 1

8 – 6 = 2

8 – 5 = 3

8 – 4 = 4

8 – 3 = 5

8 – 2 = 6

8 – 1 = 7

8 – 0 = 8

* + Child can do other tables and may use Control Chart 1 for check.

**Work of the Teacher:**

Points of Emphasis: Methodical layout of the materials, use of brown

strips, careful presentation of the blue strips.

Language: Within the lesson.

**Points of Interest:** Seeing the structure of subtraction and starting to memorize combinations.

**Work of the Child:**

Points of Consciousness: Numbers can be subtracted from each other.

Subtraction is the reverse of addition.

Control of Error: Subtraction Chart 1 can be used as a control.

**Variations:** **Extensions:**

Notes on returning the brown strips: Set box on side. Place longest strip flat in box and continue adding ‘next shortest’ strips until full, leaving an empty ‘stair’ space. This can then be filled in beginning with the shortest strip next to the second longest strip.

**Name of Activity: Multiplication with Bead Bars**

**Area:** General: Mathematics Specific: Memorization

**Materials:** A large box of bead bars including tens, a long yellow mat.

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| **Aims:** | Direct: | To further develop the Mathematical Mind, to further develop the |
|  |  | intellect and to appreciate the beauty of our number system. |
|  | Indirect: | Preparation for square root and for factoring. |
|  |  | Preparation for division by helping the child to visualize the |
|  |  | divisibility of numbers. |

**Preparation:** Numeration work, decimal system, especially using the golden bead material, addition work.

**Age:** Third year child **Presentation of the Lesson:**

**Presentation #1: Introduction (with the Table of 7)**

1. Bring the materials to the work space and unroll the yellow mat.
2. Lay 1 seven bar across the top left of the mat, horizontally.
3. Count the bar of 7 and say “7 taken one time is 7.” Lay the corresponding beads (7 bar) vertically underneath the horizontal 7 bar.
4. Lay 2 seven bars horizontally (to the right of the first 7 bar) and say “7 taken two times is 14.” Lay 1 ten bar and 1 four bar beneath the 2 seven bars.
5. Continue to lay out the table of seven, with products underneath.
6. Child can record the problems into a booklet.

**Presentation #2: All tables 1-9**

1. The child brings the box of beads to the work space and unrolls the yellow mat.
2. The child chooses his/her own number to work with and makes a table with the beads.
3. Child records the problems into a booklet.

**Presentation #3: Progressive/Cumulative Multiplication or Dynamic Multiplication**

1. As in presentation #1 use the table of 7 as an introduction to this activity.
2. Lay 1 seven bar horizontally on the mat, and say, “7 taken 1 time is 7”. Lay 1 seven bar below vertically.
3. Move the seven bar over to the right and add a second 7 bar saying, “7 taken 2 times is 14. “Make 14 with 1 gold ten bar and a four bar, and place vertically below the seven bars.
4. Continue moving the 7 bar over, adding 1 additional bar and placing corresponding bars beneath.
5. Record the answers at the end of each step.

Note: Sliding the bars over impress upon the child that multiplication is when the same number is used a different number of times.

**Presentation # 4: Multiplying by 10**

1. This activity introduces multiplying by 10; for example multiply 8 x 10.
2. Take out 10 eight bars. Place randomly on the work mat. Pick up 1 eight bar and lay horizontally across the mat.
3. Proceed to lay all of the 8 bars horizontally, one under the other.
4. Explain to the child that we are going to count the beads. When we get to 10 we are going to stop.
5. Count to 10, when 10 is reached leave finger on the bead bar and obtain 1 ten bar with the opposite hand and place vertically under the bars.
6. Continue counting to 10, stopping and obtaining 10 bar and placing vertically until all the bead bars have been counted.

7. Count by 10’s (10. 20. 30. 40. 50. 60. 70. 80). Say, “8 taken 10 times gives us 8 tens; 8 tens is the same as 80.”

1. Show the child how to record the findings. Depending on the level of the child the teacher can explain a math trick, “When we multiply a number by 10, we can add a 0 to the number. This number was 8; we add a 0, now it is 80.”
2. Invite the child to choose his/her own number to multiply by 10.

**Presentation #5: Commutative Law (Order Principle)**

1. Demonstrate by showing the child that 3 taken 7 times is the same as 7 taken 3 times.
2. Let the child think of a number. Ask how many times they would like to take that number. The child lays out the beads to make the combination.
3. Have the child lay out the beads for the reverse combination. For example; the child lays out 3 x 7, then the child works out 7 x 3. The child will see that the answer is the same. “We call this the commutative law.” (Or the Order Principle

- as the order of the numbers won’t affect the result.)

Note: If the bead bars are placed to form rectangles, child may see or be shown how the rectangles of ‘seven taken three times’ is the same as ‘3 taken seven times.’ (The orientation may be opposite.)

**Work of the Teacher:**

Points of Emphasis: Laying out the materials systematically and methodically. Language: Woven into the lessons.

**Points of Interest:** Beauty of the bead material, seeing the geometric pattern forming, seeing quantities increase.

**Work of the Child:**

Points of Consciousness: Multiplication is a form of addition and the bead

bars form rectangles.

Control of Error: Control charts, the teacher.

**Variations:** **Extensions:**

**Name of Activity: Multiplication Bead Board**

**Area:** General: Mathematics Specific: Memorization

**Materials:** Perforated board with 100 holes in ten rows of ten. In an indentation on the left side of the board, there is a little window with a slot for the insertion of number cards, a set of cards 1-10, a counter disk, 100 red beads, multiplication tables in a prepared booklet, multiplication chart #1 for checking.

**Aims:** Direct: To further develop the Mathematical Mind and heighten

awareness of the beauty of mathematics.

Indirect: To give the child further concrete experience of multiplication and to aid the child in memorization of multiplication facts.

**Preparation:** Previous operations work. **Age:** Third year child.

**Presentation of the Lesson:**

1. Child brings the material and a prepared multiplication booklet to the table.
2. The teacher chooses a table to introduce, “Let’s work with the table of 4 today.”
3. Show the child how to slot the number 4 card into the side of the board. (this represents the table the child will be working with, it is the multiplicand, the number ‘to be taken’.)
4. If not working with prepared booklets, write out the first math equation, 4 x 1=. Explain the math sentence including the multiplication sign.
5. Place the red disk in the space above the number 1 at the top of the board. This represents the number of times that 4 is to be taken.
6. Place 4 red beads under the 1 column. “4 taken 1 time equals 4.” Write the answer into the booklet.
7. Read the next equation; 4 x 2 = . Move the red disk above the numeral 2 and lay out a second row of 4 red beads. Count the total number of beads on the board. “4 taken 2 times equals 8.” Record the answer.
8. Continue in this manner until the child completes 4 x 10.
9. At some point, depending on the child, you may ask, “Do you recognize the shape that we have made with the beads?” (rectangles - can emphasize when doing 4 x 4 that it is a square.)

**Work of the Teacher:**

Points of Emphasis: Careful explanation of the exercise, ensuring that the child

lays out the same amount of beads each time and counts them to arrive at the answer, recording the answer.

Language: Within the lesson.

**Points of Interest:** Working with more abstract material, learning multiplication facts in a fun way.

**Work of the Child:**

Points of Consciousness: Numbers which are multiplied lead to a higher

number, multiplication facts can be memorized.

Control of Error: Control Chart #1.

**Variations:** **Extensions:**

**Name of Activity: Unit Division Board**

**Area:** General: Mathematics Specific: Memorization

**Materials:** The unit division board (with 81 perforations in nine rows of nine and a row of nine, There is also a row of nine circles within a green strip at the top of the board for the placement of the skittles to represent the divisor.) 9 green skittles, 81 green beads in a container, a cup to hold the beads, packet of division tables in a booklet, a red and a lead pencil.

**Aims:** Direct: To further develop the Mathematical Mind, to increase

appreciation for the beauty of mathematics.

Indirect: To help child become familiar with the ways in which numbers may be divided.

**Preparation:** Previous operations work with the decimal system. **Age:** Third year child

**Introductory Presentation using Cut Tickets (loose equations)**

1. The child selects a cut equation ticket such as 21÷7=
2. Explain that we are going to take 21 beads and share divide them between seven skittles.
3. Place seven skittles along the top of the board, stating, “We are going to divide by seven.” (This is the divisor.)
4. Have the child count out 21 beads (a container should be used to hold the “dividend”).
5. Begin sharing out the beads, explaining that “no skittle can get a second bead, before each one gets a first bead. They must each receive the same amount.”
6. Continue sharing until all of the beads are distributed, “we can see how much each bead receieved by looking down this side of the board” (indicate the numerals along the left side. These represent the answer, or quotient.)
7. Review the equation symbol if necessary and record the equation. Repeat with other examples.

Note: The above presentation is a straight forward introduction of the use of the board suitable for younger children. The following activity is a much more involved ‘research’ with emphasis on the relationship of multiplication and division combinations.

**Presentation of the Lesson:**

1. Explain to the child that there are two rules that we must remember when we work with this material:
	* the answer cannot be greater than 9
	* the remainder must be less than the divisor
2. Explain that there are 81 beads in the container. Count them to make sure. Therefore, for the first equation write 81 ÷ 9 = . Read the equation and explain the symbols.
3. Place 9 green skittles across the top row of the board. This represents the divisor, the number of times that 81 will be shared.
4. Explain that in division we share equally and that “each skittle must get 1 bead before any skittle can get 2 beads”. Place 1 bead in each hole under the row of skittles.
5. “We have enough to share again.” Continue sharing the beads in this manner until they are equally divided.
6. Record the answer: 81 ÷ 9 = 9.
7. Read the next equation: 81 ÷ 8 = . Remove the 9th skittle so that only 8 skittles are on the board.

Note: If all the beads are removed from the board each time that an equation is attempted, proceed to lay out 8 skittles and share out the 81 beads. However, if the beads remain on the board as the child progresses through the equations, proceed by removing the 9th skittle and also the beads beneath it to be redistributed, if possible.

1. Remove the beads from under the 9th skittle, as there is now no skittle

there. Each skittle has the same amount of beads, but there is a remainder. Repeat the rules of the work to the child saying that our remainder is greater than our divisor, therefore we cannot do this equation with this work. Cross out the equation on the working paper.

1. Proceed to the next equation: 80 ÷ 9 = . Take away 1 bead and replace into the container. Put back the 9th skittle.
2. We are going to divide 80 by nine. Check that 9 skittles are on the

board. Share out the beads amongst the 9 skittles. Each skittle receives 8 beads with a remainder of 8 beads. Read the equation and record the answer The remainder is written as r8.

1. Proceed to the next equation: 80 ÷ 8 = . Remove the 9th skittle and the beads beneath. Remind the child of the rule about this work and state that we cannot do this equation. (The remainder is too high.)
2. Proceed to the next equation 79 ÷ 9 = . Remove one bead and put away. Replace 9th skittle. Redistribute the beads (moving beads from bottom row first) including the ninth skittle. Answer is 8 r 7.

Note: Answers with no remainders may be underlined in red to reinforce the relationship of division to multiplication and these are the

multiplication combinations we wish the child to memorize. Example: 72 ÷ 9 = 8 (8 x 9 & 9 x 8 = 72).

**Work of the Teacher:**

Points of Emphasis: Making sure that the child understands the 2 rules applying

to this material and that the rules are repeated during the lesson.

Language: Within the lesson.

**Points of Interest:** Learning about division, using skittles.

Learning division facts.

Finding equations without remainders.

**Work of the Child:**

Points of Consciousness: Larger numbers can be divided into smaller parts. Control of Error: The teacher or control charts.

**Variations:**

**Extensions:** - Child uses prepared booklets of all division tables without remainders. Several examples may be combined on one page with decorative separations between tables. (Child may remove beads and ‘count backwards’ to arrive at new lower dividend.):

pg. 1: 81 ÷ 9 = pg. 2: 63 ÷ 9 =

^^^^^^^^^^^^ 63 ÷ 7 =

72 ÷ 9 = ^^^^^^^^^^^^

72 ÷ 8 = 56 ÷ 8 =

^^^^^^^^^^^^ 56 ÷ 7 =

64 ÷ 8 = ^^^^^^^^^^^^

54 ÷ 8 =

- Child works with box of loose tickets for isolated equations.

**Memorization: Addition Charts**

Addition Chart (control chart of tables)

A rectangular board divided into eighty-one small rectangles. On this board the complete addition table is written in sequence, one equation to a rectangle, from 1 + 1 = 2, to 9 + 9

= 18. The addends are printed in black and the sums are printed in red. The child uses this chart as a control chart.

Addition Chart 2 (control chart with duplicates removed)

A rectangular board divided into one hundred fifty-three small rectangles (nine horizontally and seventeen vertically). The board eliminates duplication by including only one pair of commutative addends. Equations with the same answer appear in the same row horizontally. As with Chart 1, the addends are printed in black, but here the smaller addend always appears first. The sums are printed in red.

The child also uses this chart as a control chart, when she understands that addition is commutative. To check a problem like 9 + 5, she must be able to reverse the order of the addend and find 5 + 9 on the chart.

Addition Chart 3 (complete finger chart)

A square board divided into one hundred small squares. The top row of squares is blue and has the numerals from 0 to 9. The column on the far left is red and has the numerals from 1 to 9 (preceded by 0 in the top left corner). Each remaining square is white and contains the sum of a horizontal (blue) addend and a vertical (red) addend. The child uses this chart to practice addition.

Procedure of use:

Have a box of prepared addition tickets (made on red paper, because addition is red) and have the child choose a ticket (i.e., 2 + 7 = ). The child places her right index finger on the numeral 2 on the top of the chart (blue) and her left index finger 7 at the left of the chart (red). She slides one finger down the column and the other across a row until they meet. The sum lies in the square where the column and row intersect (this time it intersects on the numeral 9).

Addition Chart 4 (half finger chart)

A square board divided into nine rows and ten columns. The column on the far left is red and has the numerals 1 to 9. The horizontal rows form a stair. The bottom row contains the numerals 9 through 18; the next row, 8 through 16; then 7 through 14; all the way to the numerals 1 and 2 in the top row. The child uses this chart to practice addition.

Procedure of use:

Have the child choose a ticket, read it, write down the equation. Have the child place index fingers on both addends. Place right index finger on the smaller addend in the red column and her left finger on the larger addend. She moves her right index finger to the end of its row and her left index finger until it is below her right index finger in the same column. Then she moves her right index finger down to meet the left, which should be resting on the sum.

Addition Chart 5 (‘hopping’ chart)

This chart is similar to chart 4, with the numerals 1 to 9 in a column on the far left and the numerals 1 to 18 diagonally, but all the spaces in-between are blank. The child uses this chart to practice addition.

Procedure of use:

Have the child choose a ticket, read it, write down the equation. She places one finger on each addend in the red column and moves each finger across to the end of the row. Then she moves her fingers diagonally across to the end of its row. She then ‘hops’ each finger towards each other, equal spaces. The answer is in the middle of the two fingers.

Addition Chart 6 (blank chart or ’bingo’ chart)

This chart is identical to chart 3 except that the squares for the sums are blank. Accompanying the chart is a red wooden box with one printed tile for each square on the board. The child uses this chart to practice and to confirm knowledge of addition facts.

Procedure of use:

Child picks a ticket and reads it. Fingering is the same as in chart 3; the check is also chart 3. Put the appropriate tile on the intersection of the row and column. This is how we record the answer instead of writing it down.

**Memorization: Multiplication Charts**

Multiplication Chart (control chart of tables)

A rectangular wooden board divided into one hundred small rectangles. On this board the complete multiplication table is written in sequence, one equation to a rectangle, from 1 x 1 = 1, to 10 x 10 = 100. The multiplicands and multipliers are written in black, the products in red. The child uses this chart as a control chart.

Multiplication Chart 2 (control chart with duplicates removed)

This chart is similar to multiplication chart 1, but commutative equations have been eliminated. Equations with the same multiplier appear in the same column. Squares of numbers form the diagonal. As with multiplication chart 1, multiplicands and multipliers are in black, products in red.

The child also uses this chart as a control when she understands that multiplication is commutative. To check a problem like 6 x 3, she must be able to reverse the order of the addend and find 3 x 6 on the chart.

Multiplication Chart 3 (complete finger chart)

A square wooden board divided into one hundred small squares. The top row of squares is blue and contains the numerals 1 to 10. The column on the far left is red and contains the numerals 2 to 10 (preceded by 1 in the top left corner). Each remaining square contains the product of a multiplicand (red column) and a multiplier (blue row). The child uses this chart to practice multiplication.

Procedure of use:

Have a box of prepared multiplication tickets (made on yellow paper, because multiplication is yellow) and have the child choose a ticket (i.e., 3 x 7 = ). The child places her left index finger on the multiplicand (3) in the red column and her right index finger on the multiplier (7) in the blue row. She slides one finger down a column and the other across a row until they meet. The product lies in the square where the column and row intersect (in this example, 21).

Multiplication Chart 4 (half finger chart)

This chart is similar to multiplication chart 3 except that commutative products (those above and to the right of the ‘squared’ products along the diagonal) have been eliminated. The child uses this chart to practice multiplication.

Procedure of use:

Have the child place her right index fingers on both addends. Place right index finger on the smaller factor in the red column and her left finger on the larger factor. She moves her right index finger to the end of its row and her left index finger until it is below her right index finger in the same column. Then she moves her right index finger down to meet the left, which is resting on the product.

Multiplication Chart 5 (blank chart or ’bingo’ chart)

This chart is the same as chart 3, except that all of the squares for the products are blank. Accompanying this chart is a yellow wooden box with one printed tile (product) for each space on the board. The child uses this chart to practice multiplication and to demonstrate proficiency with multiplication combinations.

Procedure of use:

Have the child pick a ticket and read it. The child fingers the board as before with chart 3, placing product tiles on the board to complete the chart. Check with the control chart.

**Memorization: Subtraction Charts**

Subtraction Chart 1 (control chart)

A rectangular board divided into one hundred sixty-two small rectangles (nine vertically and eighteen horizontally). The subtraction tables are arranged on this board in the shape of a triangle. The table includes minuends from one to eighteen, differences from zero to nine. The minuends and subtrahends are printed in black, the differences in red. The child uses this chart as a control chart.

Subtraction Chart 2 (complete finger chart)

A rectangular board with chart is shaped like a parallelogram. The ten squares in the top row are red and (from left to right) contain the numerals 18 through 9 in descending order. The squares in the diagonal line forming the right border of the chart are red. They contain the numerals 1 through 9.

The squares in the diagonal line forming the left border are blue. They also contain the numerals 9 through 1 in descending order, but a minus sign precedes each numeral. Each remaining square contains the answer for a problem composed of one red minuend and one blue subtrahend. The child also uses this chart to practice subtraction.

Procedure of use:

Have a box of prepared subtraction tickets (made on green paper, because subtraction is green). The child places her right index finger on a red minuend and her left index finger on a blue subtrahend. She slides one finger down and the other across until they meet. The answer to her problem lies in the square where the column and row intersect.

Subtraction Chart 3 (blank chart or ‘bingo’ chart)

This chart is used in the same manner as Addition chart 6, placing answer tiles on the board. The child uses this chart to practice subtraction and to verify knowledge of subtraction facts.

**Memorization: Division Charts**

Division Chart 1 (control chart)

A long rectangular board divided into squares. Across the top is a blue\* strip divided into thirty-six squares. These squares are printed with all the evenly divisible numbers (representing single digit factors only), beginning on the left with 81. Divisors are printed to the left of each horizontal row along with a division sign, beginning with 9 in the upper left and continuing to 1 to the left of the bottom row. The child uses this board to practice division.

Procedure of use:

Have a box of prepared division tickets (made on blue paper, because division is blue). The child places her right index finger on a dividend (numbers along top of board) and her left index finger on a divisor (numbers to the left of each row). She slides her right finger down a column and the other across a row until they meet. The answer (quotient) lies in the square where the column and row intersect.

Division Chart 2 (blank chart)

This chart is identical to division chart 1 except that all of the squares for quotients are blank. Accompanying this chart is a blue wooden box with one printed tile for each quotient. The child uses this chart to practice division and to verify knowledge of division facts.

Procedure of use:

The procedure is as with the first chart, with the child placing the quotient tiles within the appropriate squares on the board.

(\*Those numbers along the top blue row which are prime numbers - 1, 2, 3, 5 and 7 - are printed in white squares.)

Decimal System Passage to Abstraction

The Stamp Game

Word Problems

Dot Game

Small Bead Frame Large Bead Frame

**Name of Activity: The Stamp Game**

**Area:** General: Mathematics

Specific: Passage to Abstraction

**Materials:** Small colored tiles or stamps; green with 1 written on them to represent units, blue with 10 written on them to represent tens, red with 100 for hundreds and green, again, with 1000 for thousands; 29 skittles, 1 large green and 9 each of red and blue, 10 small green skittles to represent numerical categories. A few red, blue and green discs to represent decimal system categories; their size should be that of the base of the large skittle. A pencil and paper. It is optional to have a small set of number cards available.

**Aims:** Direct: To further develop the child's mathematical mind and appreciation

of mathematics.

Indirect: To assist the child in deepening their understanding of operations.

**Preparation:** All previous work with the decimal system, research of 10 with the snake game and the process of exchange.

**Age:** Third Year child **Presentation of the Lesson:**

**Presentation #1: Introduction**

1. Bring the stamp game to the work space and the Introduction Tray for the decimal system.
2. Demonstrate the equivalency between the bead material and the stamps:
	* one unit bead is the same as one unit stamp (green)
	* one ten bar is the same as a one ten stamp (blue)
	* one hundred square is the same as one hundred stamp (red)
	* one thousand cube is the same as one thousand stamp (green)
3. Replace the decimal system materials onto the tray and return to the shelf.

**Presentation #2: Static Addition**

1. Bring the stamp game, a pencil and a math booklet to the work space.
2. Write down an addition problem involving 4 digit numbers, when added together do not involve any carrying or exchanging.

Example: 2,134

 + 3,241

1. Read the problem and if necessary, explain the addition sign.
2. Lay out the stamps corresponding to the top number starting with units.
3. Lay out the stamps corresponding to the second number in columns underneath each category of the first number.
4. Begin the addition process, slide all the unit stamps to the bottom of the mat, count and record the answer.
5. Slide down each category, counting the total number of stamps and recording the answer before moving to the next category.
6. Summarize the lesson by reading back the equation and the answer that has been recorded. Invite the child to do another problem.

**Presentation #3: Dynamic Addition**

1. Write out two numbers for addition, ensuring that exchanges will be involved in several categories.

Example: 4566

 + 2554

1. Lay out the stamps for the first number. Lay out the stamps for the second number ensuring there is adequate space between the top and bottom number and the columns of stamps. (Align tops of columns.)
2. Begin the addition process starting with the units. Slide down the units and count. When you reach 10 stop and ask the child what we know about 10 units?

10 units are the same as one ten.

1. Exchange the 10 units for one ten stamp and add this stamp above the tens category. Record the answer for the remaining units. In this instance, a green disc is used as a zero to “hold the unit’s place.”
2. Continue adding each category, making exchanges as necessary and recording the answer before proceeding to the next category.
3. Summarize the lesson by reading back the equation and the answer that has been recorded. Invite the child to do another problem.

**Presentation # 4: Static Subtraction**

1. Write out a static problem and review it with the child, reviewing the subtraction sign.

Example: 9556

- 6432

1. Lay out the stamps corresponding to the first number. Point to the units place of the equation and say, “This says that we have to take away 2 units from our number.”
2. Slide 2 units to the bottom of the table. Repeat for each category, sliding down the stamps that are being taken away.
3. “How much do we have left?” Count each category, starting with units and record the answer.
4. Summarize the lesson by reading back the equation and the answer that has been recorded. Invite the child to do another problem.

**Presentation #5: Dynamic Subtraction**

1. The layout as for static subtraction. Example: 9556

 - 6837

1. When an exchange needs to be made, move the stamp in front of the supply box, count out 10 stamps and make the exchange.
2. Summarize the lesson by reading back the equation and the answer that has been recorded. Invite the child to do another problem.

**Presentation #6: Static Multiplication**

1. Write out a static problem and review it with the child, reviewing the multiplication sign.

Example: 2313 x 3

1. Lay the number out 3 times, leaving space between rows and columns so that the child can clearly see the amount and how many times it has

been taken. Numbers are placed in vertical columns with units on right.

1. Slide the units together, count and record the answer. Slide the tens together, count and record the answer. Slide the hundreds together, count and record the answer.

Slide the thousands together, count and record the answer.

1. Summarize the lesson by reading back the equation and the answer that has been recorded. Invite the child to do another problem.

**Presentation # 7: Dynamic Multiplication**

1. Write out a dynamic problem and review it with the child, reviewing the multiplication sign.

Example: 2596 x 2

1. Set out the stamps corresponding to the equation. In this example, lay out 2596 two times (one above the other.)
2. Slide the units together. Count, stopping at 10 and pause for the child to respond that we need to exchange. Exchange 10 units for one ten and add the ten stamp above the ten category.
3. Continue multiplying, counting and making exchanges as necessary.
4. Summarize the lesson by reading back the equation and the answer that has been recorded. Invite the child to do another problem.

**Presentation # 8: Static Division**

1. Skittles are required for this lesson. Write an equation; for example: 3693 ÷ 3 =.
2. Read the equation, saying that this tells us that we need to share 3 times, therefore we need 3 skittles.
3. Lay out 3 skittles horizontally, leaving space between each skittle.
4. Lay out the stamps for the large number, stacking the stamps for each category.
5. Begin with the thousands, sharing the stamps equally between the 3 skittles. Continue with the other categories, exchanging as needed and placing stamps in a continuous vertical column beneath the skittles.
6. Summarize the equation emphasizing that the answer is what one skittle gets.

**Presentation # 9: Dynamic Division**

1. Write down an equation; for example; 3658 ÷ 2 =
2. Point out to the child that our equation is telling us to share 3658 two times. Set out 2 skittles.
3. Lay out the stamps corresponding to the large number, stacking the stamps for each category.
4. Start to share out the quantity between the two skittles, starting with the thousands. Three thousands can be shared equally one time. Exchange the remaining thousand for 10 hundreds. Add the 10 stamps to the top of the category.
5. Continue dividing and making exchanges when necessary.
6. Summarize the equation emphasizing that the answer is what one skittle gets.

**Work of the Teacher:**

Points of Emphasis: Precise and clear layout of the material. Reviewing

the symbols for the operations and summarizing the process at the end of the lesson.

Language: Built into each presentation.

**Points of Interest:** Discovering a new material, using stamps, sliding the stamps and exchanging.

**Work of the Child:**

Points of Consciousness: Child can see operations visually. Control of Error: The teacher.

**Variations:** **Extensions:**

**Name of Activity: Dot Game**

**Area:** General: Mathematics

Specific: Passage to Abstraction

**Materials:** A reusable surface or sheets of paper that have a grid sectioned into columns headed 10,000, 1,000,100,10, 1. The columns are divided into small squares so there are ten in each horizontal row. At the foot of each column are two spaces, the upper one for indicating the “carrying process” with dots and the lower one for the result. A black pencil, a colored pencil such as orange or purple (not hierarchical colors of green, blue or red).

**Aims:** Direct: To further develop the mathematical mind and to appreciate the

complexity and simplicity of our numerical system.

Indirect: To focus the child’s attention on the mechanism of carrying. To prepare the child for abstract addition.

**Preparation:** Previous operations for memorization work, previous work with the decimal system and equivalencies.

**Age:** Third Year Child. **Presentation of the Lesson:**

**Presentation #1: Addition**

1. This is more abstract work because there are no manipulatives.
2. Explain the dot game paper, explaining the categories, including the 10,000 category.
3. Tell the child that we are going to do addition work, write out the equation on the right side of the dot game paper:

2319

4122

1956

+ 3278

1. Start with the units of the first number: “How many units are in the first number?” “9”. Start at the left side of the units column and make 1 dot per square corresponding to the units in the addend. Move to the ten category of the first number and record amount. Next, record in dots the hundred and thousand categories.
2. Start recording the second addend, as above. beginning with the units. Starting where the previous set of dots finished.
3. Repeat process for third and fourth addends.
4. Show the child how to compute the answer. Starting with the units, count to 10. Strike through this row of ten, then place one dot at the top left corner of first recording space beneath the grid for units (this shows that a ten is to be carried up to the tens category for the group of ten units just crossed through.)
5. Continue to count to ten in the next row of the units column, striking through the row and making a second dot in the space below the grid as before.
6. Finish counting the units in the final incomplete row, recording this number at the bottom of the page as the answer for the units.
7. Using the colored pencil, transfer the two dots to be carried over from the units column into the next unused boxes in the tens grid.
8. Count the tens as with the units, crossing out complete rows and placing dots for each beneath the grid. Record the remaining tens in answer space at the bottom of the page.
9. Transfer ‘carried’ dots to next space in the hundreds grid, using the colored pencil.
10. Continue in this manner with hundreds and thousands, striking out, recording, and ‘carrying’ as before. Record answer in designated boxes.
11. The answer recorded at the bottom of the page is 11,675. Read the equation

and transfer the answer below the column of figures on the right side of the paper.

**Presentation #2 (adding in columns);**

1. Place all of the dots from one category (the units). Count and cross out each group (row) of ten, placing a dot for each in the recording space beneath the grid as before. Record the amount of remaining units in the bottom space of the units column.
2. Using the colored pencil, transfer the dots from beneath the units grid up to the tens category. Begin at the top left of the tens row.
3. Continue by recording in lead pencil each of the tens quantities. Next, strike through each complete row of ten, placing necessary dots beneath the tens grids. Record the tens answer in the bottom space and bring exchanged dots into hundreds grid using the colored pencil.
4. Repeat for remaining categories until finished. Read and transfer sum to side of page, beneath column of addends.

**Work of the Teacher:**

Points of Emphasis: Explaining the ten thousand category, showing the

carrying process.

Language: Language is within the lesson.

**Points of Interest:** The dot game is fun! Counting in sets of 10, carrying dots over to the next category.

**Work of the Child:**

Points of Consciousness: Numbers can be represented in different ways, ten of one

category is the equivalent of the next higher category.

Control of Error: The teacher.

**Variations:** **Extensions:**

**Name of Activity: Small Bead Frame**

**Area:** General: Mathematics

Specific: Passage to Abstraction

**Materials:** A stand up wooden frame with four horizontal wires, each containing ten colored beads, beginning with green for units on the top wire, followed by blue for tens, red for hundreds, and on the bottom row, green for units of thousands. The left edge of the frame has printed the value of each place and is painted white for the first three categories (units family), followed by gray for the thousands place.

A paper with vertical lines in hierarchical colors for recording quantities. The value of each place is labeled on the top of the paper by families (“.....of units” and “.....of thousands”). Note - when recording on the paper (or ‘modules’) the numerals should be confined within the spaces.

**Aims:** Direct: To further develop the mathematical mind and to appreciate the

complexity and simplicity of our numerical system.

Indirect: To focus the child’s attention on the mechanism of exchanges.

To transition the child into abstract addition.

**Preparation:** Previous operations for memorization work, previous work with the decimal system materials and equivalencies.

**Age:** Third Year Child. **Presentation of the Lesson:**

**Presentation #1: Introduction**

1. Introduce the frame. Relate it to the other materials: the beads are the same color as the stamps in the Stamp Game and the number cards.
2. If necessary, give a three period lesson on the categories to familiarize the child with this material. You can introduce the hierarchy of numbers and talk about the families.

**Presentation #2: Counting**

1. Have the child count the unit beads. Remind him that ten of one category is the same as one of the next category.
2. Show how to exchange on this material: slide all the unit beads back to the left, then slide one ten bead to the right.
3. Have the child count the tens row: “One ten, two tens, three tens, etc.” (You may also count as: “Ten, twenty, thirty...,” etc.)
4. Show how to exchange 10 tens for 1 hundred. Then count hundreds and exchange 10 hundreds for 1 thousand.
5. Invite the child to count them again exchanging by himself.

**Presentation #3: Notation without Zeros**

1. Have the child count one unit, sliding it to the right.
2. Show the child how to write the numbers on the special bead frame paper. “You write ‘1’ on the green line labeled units.” (The numeral is recorded within the first space between the horizontal guide lines. A slanted stroke will be easier to read than one that is vertical.)
3. Have the child count the next unit bead and slide it to the right.
4. “You write ‘2’ below the ‘1’”. Continue in this manner to ‘9’.
5. When you get to 10 stop - slide all the unit beads back to the left.

Slide one blue 10 bead to the right. “Ten units are the same as one 10.”

1. Write ‘1’ on the blue line (without the 0) in the space below the ‘9’.
2. Continue writing ‘2’ for twenty, ‘3’ for thirty, etc., as the child counts.
3. When you have exchanged 10 tens for 1 hundred, move to the red line and write ‘1’ on the red line meaning one hundred. Continue to ‘9’ (hundred).
4. After exchanging 10 hundreds for 1 thousand, write ‘1’ on the next green line as the child counts ‘1’ thousand.
5. Invite the child to count and write the numbers several times.

**Presentation #4: Notation with Zeros**

1. Have the child count and write the units as in the above presentation.

2 When the child gets to 10, show that he writes the ‘1’ on the blue line as

before and the ‘0’ on the green line. This means there is 1 ten and no units (10).

1. Have him write through ‘90’ as he slides the corresponding beads.
2. Make an exchange of 10 tens for 1 hundred and show that the ‘1’for one hundred is written on the red line; the ‘0’ for no tens on the blue line; and the ‘0’ for no units on the green line. (100)
3. Continue counting and recording the hundreds until 900.
4. Exchange 10 hundreds for 1 thousand and show the child how to record this on the paper with ‘1’ on the green line for ‘units of thousands’ and zeros on each of the lines for the hundreds, tens and units (1000).

**Presentation #5: Formation of Numbers**

* 1. (slide beads first, then record number)
		1. Turn the paper over and tell the child, “We are going to make numbers.” (May begin with single digit, then two-digit numbers, etc.)

Slide some beads of each category to the right.

* + 1. Have the child count the units and write that number on the green line in the top space.
		2. Child repeats with tens, hundreds and thousands, recording amounts on corresponding lines in top space. Child reads number.
		3. Give a few more examples, having the child leave a few lines between each number for isolation and decoration.
	1. Adult writes number on paper. then the child forms on the frame with beads, beginning with units and then progression with tens, hundreds, and thousands. Read number and repeat with other examples. (Include some with zeros.)
	2. The child writes his own number and then forms it on the frame. Encourage repetition.

Note: The child can go right on to addition. They must be able to form complex numbers first. Through these first exercises, the child gains an understanding this it does not matter which number is written down first as long as it is in the correct position on the line. It is the place that the figure holds that determines its value.

**Exercise 1: Static Addition (This first addition is without paper.)**

* + 1. Tell the child that we are going to add.
		2. Ask the child to slide (5) units, (3) tens, (1) hundreds, and (2) thousands to the right. (amounts depicted in accompanying illustration)
		3. Have the child read this number. Do another number in the same manner, keeping them slightly apart on the wire. (2 units, 4 tens, 3 hundreds, 1 thousand)
		4. Have the child push the two parts together and count them. This is the answer.
		5. Do several examples of static addition until the child is secure with it and then move right on to dynamic addition.

**Exercise 2: Dynamic Addition (written down)**

* + - 1. **Addition by Addend**
				1. You write the example, ask the child to help you think of the addends.

Example: 4526

+ 1978

* + - * 1. Ask the child to slide the beads for the first addend. Slide the units first, then the tens, hundreds and thousands.
				2. Do the second addend, showing how to carry by counting the last bead on the wire and saying/singing or chanting that number while sliding all the unit beads back to the left and sliding one ten to the right...

“one, two, three, fou....fou.....fou....fou.....four.”

* + - * 1. Continue counting the second set of units (8) in the second addend again from the ‘remembered’ number: “....four....five, six, seven, eight”.
				2. Repeat in the same way with the tens. In this example, there will be a complete set of 10 tens to exchange with no more to be ‘remembered’.
				3. Next, continue with the hundreds, exchanging on “fou....fou…four....” and continue counting to nine (with 5 hundreds in the answer).
				4. Repeat for the thousands. When finished, have the child count the beads on the right side of the frame and record the answer.
				5. Invite the child to do many examples and to decorate his work.
			1. **Adding One Category at a Time**
				1. Ask the child to think of two addends and write them down. (For comparison, the same example is being used as in Presentation A.)
				2. Show the child how to add just the units first: count and slide the units from the first addend (6), and then count and slide the second set of units (8), exchanging when necessary (after 4 of the 8). Have the child read and record the answer (4) on the paper in the units place.
				3. Do the tens next. Slide the tens beads for the first addend (2), adding them to

the 1 ten ‘carried’ from the units. Slide the ten’s beads for the second addend (7).

1. Exchange for 1 hundred, writing zero in the ten’s place in the answer.
2. Continue in the same way for the hundreds and thousands. Record and read the finished equation.
3. Invite the child to do several examples and to decorate his work.

Extension I:

After the child has much experience with dynamic addition, give him an example that involves carrying to each category and with an answer of 1000.

Examples: 996 999 992

+ 4 + 1 + 8

Extension II:

Give the child long columns of numbers to add, keeping the thousands low so as to not reach 10.

Extension III:

Give the child an example with zero in the addend.

**Exercise 3: Static Subtraction (without paper)**

1. Give the child a minuend. He makes it on the frame. Example: 4769
2. Dictate a subtrahend by category, starting with units. Example: 1342
3. Show the child to count that many beads from the minuend and slide them back to the left.
4. Repeat for the other categories. The remaining beads are the answer.
5. Do a few examples with the child, then move right on to dynamic subtraction.

**Exercise 4: Dynamic Subtraction (with recording)**

|  |  |  |
| --- | --- | --- |
| 1. Ask the child to write the minuend. | Example: | 4526 |
| You write the subtrahend. | Example: | 1978 |

1. Have the child make the minuend on the frame.
2. Starting with the units, subtract showing borrowing: count the unit beads on the right, returning them to the left....”one, two, three, four, five, sic...sic...sic...six.
3. Remember the number of the last counted bead (6), as in addition. Slide one ten bead from the right back to the left. Push all the units to the right and finish subtracting... “7, 8”.
4. Continue subtracting 10’s, 100’s and 1000’s in this manner, borrowing when necessary.
5. The child counts the beads on the right and records the answer.
6. Invite the child to do another problem. He can write both the minuend and the subtrahend.
7. Encourage repetition and invite the child to decorate his work.

Note: As the child is able, you may increase the level of challenge, as in addition, by including first one, and other zeros in the minuend. This is a challenging activity and allows the child opportunity to verbalize various equivalencies and to demonstrate this in the multiple exchanges required.

**Work of the Teacher:**

Points of Emphasis: Recording and formation of large numbers.

Use of zero within large numbers as ‘place holder’. ‘Remembering’ exchanged number.

Language: Language is within the lesson.

**Points of Interest:** It is possible to carry out dynamic operations with large numbers on this material with a limited supply of beads.

**Work of the Child:**

Points of Consciousness: Ten beads within each category are sufficient to carry out

exercises of addition and subtraction, using the exchange process.

Control of Error: The teacher.

**Name of Activity: Large Bead Frame**

**Area:** General: Mathematics

Specific: Passage to Abstraction

Materials: A stand up wooden frame similar in appearance to the small bead frame, with a total of seven wires representing categories from units to units of millions. The gray portion on the left side of the frame is expanded to include units, tens and hundreds of the thousands family, followed by black for the units of millions.

A paper with vertical lines in hierarchical colors for recording quantities up to units of millions. The value of each place is labeled on the top of the paper by families (“.....of units”; “.....of thousands”; and “...of millions”).

**Aims:** Direct: To further develop the mathematical mind and to appreciate

the complexity and simplicity of our numerical system.

Indirect: To focus the child’s attention on the mechanism of exchanges.

**Preparation:** Previous operations for memorization work, previous work with the decimal system materials and equivalencies.

**Age:** Third Year Child.

Note: The large bead frame is used to show multiplication only. The child may do addition and subtraction, but it is not shown.

**Presentation of the Lesson:**

**Presentation #1: Introduction**

Introduce the new categories of 10,000; 100,000; and 1,000,000 - said as, “tens of thousands and hundreds of thousands”.

**Presentation #2: Counting**

Procedure is the same as the small bead frame. Have the child count the beads starting at units. Guide her in changing to the next category.

**Presentation #3: Notation without Zeros**

Same as with the small bead frame, relate the color of the beads of the new categories to the color of the lines the numerals are written on.

**Presentation #4: Notation with Zeros**

Same as the small bead frame, the child counts and writes the number.

**Presentation #5: Formation of Numbers**

(Note: Reading these numbers takes much practice.)

* 1. The adult makes a number with the beads; the child counts and writes it on the paper.
	2. The adult writes the number; the child makes it with the beads.
	3. The child writes her own number and makes it on the frame.

**Exercise 1: Multiplication by Units**

(The child performs the multiplication on the frame.)

1. You write a four-digit multiplicand; ask the child how many times she wants to multiply this number (suggest a low multiplier for the first few times).

Example: 4962 x 3

1. Start with the units. Show to slide the number of units in the multiplicand (2) as many times as the multiplier says (3). Leave a small space between them.
2. Push the beads together and count them for the answer.
3. Continue in the same manner for the 10’s, 100’s and 1000’s; exchanging as necessary. (This work involves counting how many times you take the number and remembering the number when carrying.)
4. Record the answer, beginning with units. then read the completed equation. Do several examples with the child.

**Exercise 2: Multiplication by Units**

(The child performs the multiplication in her head and places the results on the frame.)

1. Invite the child to write a large multiplicand and a one-digit multiplier.

Example: 5892 x 4

1. Ask the child how many units there are and how many times we’re going to take that number. (“Two units taken four times.”)
2. Instead of working this out on the frame, ask her to do it in her head (or “What is 2 x 4?”....or, “What is 2 taken 4 times?”)
3. Have the child slide this many beads (8) to the right and to record this on the paper.
4. Continue this process pointing out in each category that you are actually multiplying by 10’s, or 100’s, or 1000’s

(For the tens place, the child asks herself how much 9 x 4 is, but that this is

9 tens, or 90, taken 4 times. This is 36 tens, which will be placed on the frame as 6 tens and 3 hundreds.)

1. Child records answer after each category and reads problem when finished. (Watch for multiple exchanges in which child may need assistance.)

**Work of the Teacher:**

Points of Emphasis: Recording and formation of large numbers.

Use of zero within large numbers as ‘place holder’. ‘Remembering’ exchanged number.

Language: Language is within the lesson.

**Points of Interest:** It is possible to carry out dynamic operations with large numbers on this material with a limited supply of beads.

**Work of the Child:**

Points of Consciousness: Ten beads within each category are sufficient to carry

out the exercises on the frame, using the exchange process.

Large multiplication is based on the basic multiplication tables/facts, but within the appropriate place or category.

Control of Error: The teacher.

Other Areas of Study

in Mathematics

Exercises with Fractions

Notes on Geometry

FRACTIONS Introduction, Terminology and Equivalence

PRELIMINARY INTRODUCTION:

Material: The fraction squares, the fraction circles and the fraction triangles.

After the child has become familiar with the Geometry Cabinet, the Constructive Triangles and the Metal Insets, she is introduced to the Small Metal Insets (also referred to as the Fraction Insets or Divided Metal Insets.)

The child is shown that she may take any of the insets and carry these to a work space. Show the child how to make a design with

the various parts of the insets. Encourage the child to experiment and make any design she wishes.

Show how to put the material away: In order to return each piece

to its proper inset, the child is shown how to measure one piece against a piece already in the frame.

Extension: Show the child how to transfer the design onto paper. First, the

design is made on the table, then the pieces, one at a time, are taken out of the design and traced onto the paper and then the pieces are returned to the design.

(Note: This preliminary work is important preparation before the children are introduced to the fraction exercises.)

PRESENTATION A: Fraction Terminology Material: The divided circles

Set out all the circle insets in sequence.

Explain the term “fraction”: a fraction is a part of something. (A fraction of an apple is part of an apple, etc.)

Explain that so far numbers have been family names: units, tens, hundreds and thousands. In fractions there are families of 2, 3, 4, 5, etc. Point out the divided circles giving their family name: the circle with two pieces is referred to as the family of 2, the circle with 3 pieces is referred to as the family of 3, etc. Explain that the number which indicates the family of a fraction is called the denominator.

Show how to write the denominator. In the family of 3, the denominator is 3. The 3 is written below the line. Give many examples of writing denominators allowing the child to participate.

When the child has an understanding of denominator, introduce the term numerator.

Explain there can be any number of parts in a fraction.\* The number telling how many parts of a family there is called the numerator.

Show how the numerator is written above the line: 2 parts of a fraction; 5 parts of a fraction; etc.

Do the second period using the insets: “show me 3 of the family of 4”, “take out 2 of the family of 3”, etc. Show how each is written: 3/4; 2/3, etc.

Invite the child to write the fraction and to draw the corresponding insets on paper.

(Note: when the numerator and denominator are the same, the

fraction is equal to the whole number 1. If the numerator is larger than the denominator, this is referred to as an improper fraction and can be converted into either a whole number or a mixed number - a whole number and fraction amount together. It is not necessary to explain this to the child at this time.)

Terminology of Fraction Families:

Teach the correct terminology of fractions: if a circle or an object is divided into two parts, each piece is called one half, in the family of 3 each piece is called a third, etc.

Do a second and third period: “Show me 2/3.” And “What is this?”.

EXERCISE I: Prepared Tickets

Introduce the prepared tickets/labels for each part of the divided circles: 1, 1/2. 1/2, 1/3, 1/3, 1/3, 1/4, 1/4, 1/4, 1/4, etc.

Name each fraction piece and place the appropriate label on it.

PRESENTATION B: Introduction to Equivalencies

(Note: The child’s work with equivalencies is a very important part of the fraction work. The manipulation of the fraction insets and experimentation with equivalencies lay the groundwork for the child to find the lowest common denominator.)

Take out the whole circle/family of 1 and try to fit parts from other families into the frame, i.e., 1/2 + 1/2, or 1/2 + 1/4 + 1/4, etc.

Take 1/2 + 1/2. Each time say, “One whole equals 1/2 + 1/2.”

This time take the circle divided in half. Take out 1/2 and fill the space with various combinations, verbalizing each equivalency, i.e., 1/2 = 1/4 + 1/4. Continue with the circle of 3, making equivalencies of one third.

EXERCISE I: Writing Equivalencies

Show how equivalencies are written. After each example, show in writing all equivalencies for each family. Go from 1 = 1/2 + 1/2 to all other combinations of fractions making up one whole. Then proceed to 1/2 =.

DIRECT AIM: Same as for math.

INDIRECT AIM: The child is given the experience of fractions and equivalencies through experimentation.

AGE: 5 – 5 1/2

GEOMETRY

Geometry may be defined as the study of form. To make the subject of mathematics understandable, it is essential that the three areas involved should not be divided. These are: arithmetic, algebra and geometry. They are treated as one subject in the Montessori classroom. Any piece of apparatus in one syllabus has implications for the other areas. Example: the pattern of the multiplication tables in the Bead Bar Layout visually and actually follows the same pattern as the rectangles in the Geometric Cabinet. And then you are also immediately in the realm of algebra:

a² or b ² is just as easy to understand as 5²;

and now there is an equation involved. The juniors draw on graph paper the mathematical concept realized with the beads.

It is only logical that these three are side by side. The sensitive period for geometry comes during the sensitive period for sensorial exploration. And the preparation for geometry as well as algebra comes very early.

INTRODUCTION

The child sees geometrical forms within the environment continually. Thus, what you are doing in the class is isolating what is part of his normal environment when classifying, ordering and clarifying. The child can take in geometrical terminology just as easily and naturally as everyday language. What we are doing at this stage is taking on and building up the experience of life so that the child may later build with formal geometry. Remember, the geometry vocabulary will be needed by the child later for geometry theorems.

The sensitive period for the area of language is 2.5 to 5; the language of the geometrical environment comes before the geometry cards.

MATERIAL

The Tower of Cubes, Broad Prisms, and Length Rods are preparing the child for a sense of discrimination of form. Since the size differential is regular and in a series based on 10 there is also arithmetical and algebraic preparation. All of these preparations are indirect and in the form of muscular and visual impressions.

The Geometric Solids give the child experience with solid geometric form, as well as language.

The Folding Cloths in the various divisions prepare the child for the small metal insets and work with concepts of similarity and equivalence.

The Binomial and Trinomial Cubes give the geometric impression of the algebraic formula (a + b)2 and (a + b + c) 3.

The Geometric Cabinet presents the child with the opportunity to study, observe and manipulate form - muscular memory. The triangle, square, etc., are as accurate as materialization can be. Montessori took the subject matter and put it into a material the child could feel and touch.

It is important when introducing the geometry cabinet to stay with the child long enough to know that the exercise is meaningful to them. Introduce the language as soon as they are familiar with the figure. Then the children may feel the inset with their eyes shut and give the name. The exercise then becomes a stereognostic exercise. This may have to be done very early to maintain interest, but do not start with it as it is intended to be a visual exercise. Move quickly onto the language.

The different shaped tables in the classroom again expose the child to varied forms; and certain discoveries are possible, i.e., 2 trapezoidal tables placed together form a hexagon; 2 squares form a rectangle.

Next, comes the three sets of Geometry Cards which serve to lead the mind and the eye toward the abstraction; the idea of circle, square, etc. The drawers of the cabinet, as well as the cards, bring the child to the realization of the different sizes of figures and the different forms. Also, through the repetition with the Cards, the child is gaining an intuitive knowledge. Gradually, as the child progresses through the sets of cards, he comes to the realization that the form of a geometric figure is the space enclosed by lines. This realization is acquired only by experience - using together the hand and the eye. The Cards also show the child the similarities between diverse shapes and forms and similar shapes and forms.

The Constructive Triangle Boxes go one step further in the preparation of the child for deductive geometry. The exercise comes after the child knows the names of the figures from the Geometry Cabinet. The child is also preparing for work with the theorems for area, and for the measurement of figures. The triangles help the child to evolve a geometric sense; and there is an extension of vocabulary with the names of the parts of the triangles.

The Metal Insets provide the child further experience with feeling and matching, as well as opportunities for drawing the geometric figure. The frames offer greater initial control of error and the inset figures help as something to draw around - a concrete object producing a concrete form after drawing. The insets also enable the child to acquire finer intuitive knowledge of the analytical characteristics of a geometrical figure by enabling the child to make a form by himself.

The Superimposed Geometric Figures help the child to observe finer and finer detail, the ability to recognize component parts of any one figure.

There are two sets of Geometric Language Cards (Geometry Nomenclature Material) and the process is the same as with the environment or geography three-part matching. There are also the definitions and the definition booklet. The language given is the parts of the various figures. The progression of difficulty followed is first the whole, then the details - the form, the parts of the form, the kinds of parts. Example: the triangle names, the parts of the triangle (side, angle, midpoint, vertex, etc.) and the names of angles (acute, scalene, obtuse, right).

The child may make for himself a book of various angles or other forms after he has had the language. In this way he gets used to the different geometrical instruments. As the child progresses in the area of reading, he can add definitions to his book.

Geometry Instruments can be used after some experience with the Metal Insets. A lesson may be given on how to measure with a ruler, with a tape measure or with a steel rule that pulls out. The child may then measure other objects in the room, such as furniture, materials, etc. Also, teach the construction of figures, such as triangles and squares. Tell the child to construct squares of various size sides, then measure them exactly. Draw various angles and measure them.

Later, go back to the Superimposed Geometry Figures and show them how to draw and construct for themselves figures giving the same effect. Thus, you have led the child from the realm of sensorial exploration and language to the ability to construct for themselves within the realm of geometry.

CONCLUSION

The interrelationship of all knowledge must be impressed upon the child. He must see that it is fun, it is an adventure, so he does not bring a barrier with him to the geometry work. He is merely having fun with the shapes.